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# *RAEM: version 3.0*

## FIRST REPORT

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# *Structure of the report*

The present report contains the description of the spatial general equilibrium model for the Netherlands (RAEM 3.0), which is developed by Transport and Mobility Leuven for the project financed by TNO (Netherlands).

The report stars with an overview of the old version of the model called RAEM 2.0 and illustrates the major differences between the two model versions.

It further gives the full description of the model structure and all its main components. The model description consists of multiple paragraphs devoted to the description of the particular part of the model including the behaviour of each economic agent in RAEM 3.0, external trade, interregional trade, functioning of the labour market and commuting, market clearing for goods and services, imperfect competition, generation of the passenger trips by trip purpose.

Next paragraph of the report is devoted to the description of the model development including the construction of the detailed model database, calibration technique, used to derive the coefficients of the model, and implementation of the mathematical formulation of the model in GAMS language. This paragraph contains the description of the model GAMS code and provides the model users with instructions on how to run the model and extract the model results.

Finally the report contains the bibliography of the literature related to general equilibrium modelling and new economic geography and the full mathematical formulation of RAEM 3.0.

# I Introduction

SCGE models typically are comparative static equilibrium models of interregional trade and location based in microeconomics, using utility and production functions with substitution between inputs. Firms often operate under economies of scale in markets with monopolistic competition of the Dixit-Stiglitz (1977) type. The few empirical applications of this approach are Venables and Gasiorek (1996) and Bröcker (1998). Interesting theoretical simulations with a SCGE model with a land market are found in Fan et al. (1998). These models are part of the new economic geography school (Krugman, 1991, Fujita, Krugman and Venables, 1999) and have been around for less than a decade.

The present, still young SCGE models have a sophisticated theoretical foundation and rather complex, non-linear mathematics. The latter is precisely the reason why SCGE models are able to model (dis)economies of scale, external economies of spatial clusters of activity, continuous substitution between capital, labour, energy and material inputs in the case of firms, and between different consumption goods in the case of households. Moreover, monopolistic competition of the Dixit-Stiglitz type allows for heterogeneous products implying variety, and therefore allows for cross hauling of close substitutes of products between regions.

The present report describes the SCGE model developed for the Netherlands and called RAEM 3.0. The model fits in the new economic geography theory as described in Fujita et al. (1999) and builds on models developed by Venables (1996) and Oosterhaven et al.(2001).

RAEM 3.0 model utilize the notion of the aggregate economic agent. It represents the behaviour of the whole population group or of the whole industrial sector as the behaviour of one single aggregate agent. It is further assumed that the behaviour of each such aggregate agent is driven by certain optimization criteria such as maximization of utility or minimization of costs.

The RAEM 3.0 model includes the representation of the micro-economic behaviour of the following economic agents: 15 production sectors; federal government and external trade sector.

The RAEM 3.0 model is a dynamic, recursive over time, model, involving dynamics of capital accumulation and technology progress, stock and flow relationships and backward looking expectations. A recursive dynamic structure is composed of a sequence of several temporary equilibriums. The first equilibrium in the sequence is given by the benchmark year 2006. In each time period, the model is solved for an equilibrium given the exogenous conditions assumed for that particular period. The equilibriums are connected to each other through capital accumulation. Thus, the endogenous determination of investment behaviour is essential for the dynamic part of the model. Investment and capital accumulation in year t depend on expected rates of return for year t+1, which are determined by actual returns on capital in year t.

Behaviour of the households is based on the utility-maximization principle. Household's utility is associated with the level and structure of its consumption. Each household spends its consumption budget on services and goods in order to maximize its satisfaction from the chosen consumption bundle. Utility of the household is maximized under the budget constraint, where the household's consumption spending is equal to its income minus income tax and the household's savings. Households in the RAEM 3.0 model receive their income in the form of wages, capital rent and transfers from the federal government. The governmental transfers consist of the unemployment benefits and other transfers.

The level of the unemployment benefits, received by the household, depends upon the level of unemployment associated with this particular household type. The voluntary unemployment in the economy is modelled according to the wage curve, which relates the level of the unemployment and the level of the real wages in the economy.

Behaviour of a sector is based on the minimization of the production costs for a given output level under the sector's technological constraint. Production costs of each sector in the model include labour costs, capital costs and the costs of intermediate inputs. The sector's technological constraint describes the production technology of each sector. It provides information on how many of different units of labour, capital and of the 15 commodities, traded in the economy, are necessary for the production of one unit of the composite sectoral output. Each sector in the economy may produce more then one type of commodity and the combination of these different commodities corresponds to the sectoral composite output.

The RAEM 3.0 model adopts the assumption of the average costs pricing in combination with the assumption of the Dixit-Stiglitz varieties and monopolistic competition between the firms inside each sector. Under the monopolistic competition framework, it is assumed that each sector consists of a number of identical firms, each producing a unique specification of a particular commodity. The same type of the commodity, produced by an individual firm, is slightly different from the same type of commodity, produced by other firms inside the sector.

These differences in the commodity specification give individual firms a certain monopolistic power over the consumers. Certain consumers prefer a certain specification of the commodity and, hence, they are prepared to pay a bit more for it. The monopolistic power of the individual firms results in the deviation from the marginal costs pricing rule of perfect competition. The producer prices are now equal to the sector's average production costs and depend upon the number of the individual firms, which operate on the market.

Under the assumption of imperfect competition (monopolistic competition in case of the RAEM 3.0 model) the total sectoral production costs consist of the variable costs and of the fixed costs. The sectoral variable costs are equal to the marginal output costs multiplied by the sectoral output level. The sectoral fixed costs depend upon the number of the individual operating firms and are equal to the number of firms inside a sector multiplied by the fixed costs per firm.

Zero profit condition should hold for each of the 15 sectors and it states that the variable production costs plus the fixed production costs are equal to the total revenues of the sector. The zero profit condition defines the equilibrium number of the individual operating firms in each sector. The number of firms is such that any new firm entering the market causes negative profits for all the firms in the sector. Increase in the number of the firms operating in a particular industrial or service sector is beneficial for the consumers since it provides them with better choice opportunities.

The outputs of the domestic sectors are either consumed inside the country or exported abroad. Each of the domestic sectors chooses how much of its output to sell inside the country, how much to export to the EU25 and how much to export to the rest of the world.

Domestic sales of each of the 15 types of commodities composed of the commodities produced by the domestic sectors, those imported from the EU25 and those imported from the rest of the world. According to the Armington assumption, the same type of commodity produced by the domestic sectors, imported from the EU25 or imported from the rest of the world has different specifications and, hence, cannot be treated as a homogenous good. Domestic consumers have different preferences for these three specifications and can substitute between them in case the relative prices of the specifications change. The substitution possibilities between these three commodity specifications are represented by the Armington elasticity of substitution and vary between the types of commodities. The shares in which commodity is bought from the domestic producers, from the EU25 and from the rest of the world are determine by the relative producer prices of the commodity inside the country, in EU25 and in the rest of the world as well as by the Armington elasticity of substitution.

The equilibrium prices of all commodities are defined by the market equilibrium conditions. Under the market equilibrium the sum of demands for a particular commodity is equal to the sum of its supplies. Due to the existence of unemployment the labour market is assumed to be in disequilibrium such that the equilibrium labour prices is determined by the equality between the sum of the sectoral labour demands and the total labour endowment in the economy minus the unemployment.

The model incorporates the representation of investment and savings decisions of the economic agents. Savings in the economy are made by households, government and the rest of the world. The total savings accumulated at each period of time are invested into accumulation of the sector-specific physical capital, which is not mobile between the sectors. The stock of this capital at each period of time is equal to the last period stock minus depreciation plus the new capital accumulated during the previous period of time.

The total investment into the sector-specific capital stock is spent on buying different types of capital goods such as machinery, equipment and buildings. The concrete mixture of different capital goods used for physical investments is determined by the maximization of the utility of the investment agent. This is an artificial national economic agent responsible for buying capital goods for physical investments in all the domestic sectors.

The RAEM 3.0 model incorporates the representation of the federal government. The governmental sector collects taxes, pays subsidies and makes transfers to households, production sectors and to the rest of the world. The federal government consumes a number of commodities, where the optimal governmental demand is determined according to the maximization of the governmental consumption utility function. The model incorporates the governmental budget constraint. According to this constraint the total governmental tax revenues are spend on subsidies, transfers, governmental savings and consumption.

Finally, the model includes the trade balance constraint, according to which the value of the country's exports plus the governmental transfers to the rest of the world are equal to the value of the country's imports.

Households and domestic sectors use transport services in their consumption and production activities. The passenger transportation services in the model are used for different purposes, which are represented separately. These include business, commuting, shopping, education and travel. The amount of the trips associated with each of these purposes are generated based on a set of the specific trip generation functions, which take into account separately time and monetary costs of travel as well as a set of the attraction factors for the trips.

The main differences between the old (RAEM 2.0) and the new (RAEM 3.0) SCGE model for the Netherlands include the representation of the international trade, boarder representation of the federal government, different and more detailed treatment of the passenger trips and interregional migration.

# II Overview of main features of RAEM 2.0

The following spatial effects are incorporated in the model:

1. 3 agglomeration effects:
   - Market access effect: monopolistic firms will try to locate themselves in a big market and export to small markets. This way they minimize their transport costs and are the most competitive in all regions.
   - Variety effect: Monopolistic firms will try to locate themselves in a big market with the most varieties to gain in productivity via a larger variety of intermediate inputs.
   - Cost of living effect: Goods tend to be cheaper in a region with more economic activity since consumers import less and reduce their transport costs.
2. 2 dispersion effects:
   - Monopolistic firms have an incentive to locate themselves in regions with fewer competitors to avoid strong competition.
   - Housing effect: people have a tendency to migrate to areas with little competition for land and housing.
3. Production (except transport):
   - The model incorporates the Dixit-Stiglitz monopolistic framework.
   - The production function has two levels:
     - The upper level is a Cobb-Douglas function for the choice between labour and intermediate (composite) inputs
     - The lower level is a CES function that allocates expenditure on intermediate inputs between the different varieties – note: the elasticity of substitution is taken to be equal for all sectors.
   - Firms are assumed to minimize costs which consist of a variable cost depending on the output level and a fixed part depending on the number of varieties. The proportion of labour and intermediate inputs is assumed to be the same in the fixed part as in the variable part.
   - Free entry and exit imply zero profits. Price equals average costs. The number of varieties is determined endogenously in the model.
   - The framework allows for internal economies of variety.
4. Households:
   - The households' preferences are described by a two-level utility function:
     - The upper level is an LES function that models the choice between different (composite) goods.
     - The lower level is a CES function that allocates expenditure on the composite goods between the different varieties – note: the elasticity of substitution is the same as for the production functions.

   - Household income consists of labour income and unemployment benefits.
   - Labour supply is determined completely by population – no inactive population (apart from unemployed) – leisure does not enter the utility function

5. Labour market – short run
   - No migration is possible, but commuting is.
   - Commuting between two regions depends on the transport costs, the number of unemployed people in the origin and the number of vacancies in the destination. A matching function according to Pissarides' approach is included. Two constraints need to be met:
     - supply of labour in each region (taking into account commuting to and from the region) should equal demand.
     - People commuting from a region should equal the number of people living in that region minus the unemployed living in that region.
   - The number of vacancies in a region is set such that the cost of an extra vacancy equals its benefit.
   - The model without migration implies utility differences between regions.
6. Labour market – long run
   - Migration will take place until utility is equal for all labourers, taking into account both utility from consumption and utility from living in a region.
   - Utility from living in a region depends on region specific quality factors of housing and on the number of houses per person.
   - Total population should equal the sum of population in the different zones.
7. Transport:
   - The model distinguishes three types of transport costs (that are determined exogenous to the model):
     - Commuting costs
     - Transport costs of transporting goods from the factory to the shop
     - Shopping costs (search costs for finding the best variety and bringing the goods from shop to home, or bringing the consumer to the shop)
   - Transport costs are not modelled as iceberg costs. The iceberg approach assumes that transport is produced with the production function of the good that is transported. However,
     - It is clear that this is not realistic, see e.g. services sector.
     - A reduction in transport costs may lead to a fall in sectoral production.
   - The transport costs of bringing goods to the shop are modelled as a mark-up over prices.
   - The model includes a perfectly competitive transport sector which produces transportation of goods from the factory to the shop.
   - Shopping costs are assumed to be non-monetary costs (not produced by any sector)

  - It is not clear by which sector commuting transport is produced or whether this also concerns non-monetary costs.

8. Federal government:
   - Raises income taxes
   - pays unemployment benefits
   - budget balance is assumed
9. No international trade. Closed economy.

# III Main differences between RAEM 2.0 and RAEM 3.0

RAEM 3.0 is a new model version which is developed by TML (www.tmleuven.be) for the project financed by TNO (Netherlands). The main differences of RAEM 3.0 with respect to RAEM 2.0 are:

1. The introduction of dynamics
   - The model is recursive over time.
   - The modelling of savings, capital accumulation, technological progress, stock and flow relationships and backward looking expectations is included.
   - Capital is sector-specific (not mobile between sectors).
   - The mixture of capital goods is chosen by utility maximizing investment agent (artificial economic agent)
2. International trade
   - Domestic demand is allocated to goods produced in the domestic regions, the EU25 and the ROW. Next an allocation is made between varieties produced within the regions. The formulation of an international trade is based on the Armington assumption.
   - Trade balance constraint
3. A larger role for the federal government
   - It collects taxes, pays subsidies and makes transfers to households, firms and ROW.
   - It consumes commodities – the choice is made by maximizing utility function subject to a budget constraint.
4. More detailed modelling of passenger trips, including
   - Education trips;
   - Commuting trips;
   - Shopping trips;
   - Business trips;
   - Other passenger trips;
5. Modelling of savings and investments
   - Households
   - Government
   - Sectors
6. Interregional migration
   - Migration is modelled according to the discrete choice migration generation function and split into the two steps

- At the first step a person decides whether to move from the region or not, where this choice depends upon the sector specific preferences and the difference between the average regional household's utility and the household's utility of this particular region
- At the second step the person decides to which region he is going to move, where this choice is based on the region specific preferences and the level of the household's utility in the destination region

# IV Model structure

This part of the report describes in detail the structure of the RAEM 3.0 model and its underlying theoretical assumptions. The model description includes paragraphs with the description of the behaviour of each economic agent in RAEM 3.0, external trade, interregional trade, functioning of the labour market and commuting, market clearing for goods and services, imperfect competition, generation of the passenger trips by trip purpose. The mathematical notation used for the description of the RAEM model in this chapter of the report is given in full details in beginning of chapter VII of this report.

The RAEM model includes the representation of the following commodity/sector groups:

1 Agriculture
2 Mining and quarrying
3 Manufacturing
4 Electricity, gas and water supply
5 Construction
6 Trade and repair consumer services
7 Hotels, restaurants and café
8 Transport
9 Storage and communication
10 Financial services
11 Business services, renting, real estate
12 Public administration (includes defence and collective social security)
13 Education
14 Health and social work
15 Culture, sports, leisure

Production and consumption activities in the model take place in the 40 Dutch regions corresponding to the NUTS3 classification of EuroStat, which is identical to the COROP classification of the Dutch statistical office:

1 Oost-Groningen
2 Delfzijl en omgeving
3 Overig Groningen
4 Noord-Friesland
5 Zuidwest-Friesland
6 Zuidoost-Friesland
7 Noord-Drenthe
8 Zuidoost-Drenthe
9 Zuidwest-Drenthe
10 Noord-Overijssel
11 Zuidwest-Overijssel
12 Twente
13 Veluwe
14 Achterhoek
15 Arnhem/Nijmegen
16 Zuidwest-Gelderland
17 Utrecht
18 Kop van Noord-Holland
19 Alkmaar en omgeving
20 IJmond

| | |
|---|---|
| 21 | Agglomeratie Haarlem |
| 22 | Zaanstreek |
| 23 | Groot-Amsterdam |
| 24 | Het Gooi en Vechtstreek |
| 25 | Agglomeratie Leiden en Bollenstreek |
| 26 | Agglomeratie 's-Gravenhage |
| 27 | Delft en Westland |
| 28 | Oost-Zuid-Holland |
| 29 | Groot-Rijnmond |
| 30 | Zuidoost-Zuid-Holland |
| 31 | Zeeuwsch-Vlaanderen |
| 32 | Overig Zeeland |
| 33 | West-Noord-Brabant |
| 34 | Midden-Noord-Brabant |
| 35 | Noordoost-Noord-Brabant |
| 36 | Zuidoost-Noord-Brabant |
| 37 | Noord-Limburg |
| 38 | Midden-Limburg |
| 39 | Zuid-Limburg |
| 40 | Flevoland |

## *IV.1 Households' incomes, savings and consumption budget*

The model incorporates the behaviour of one representative regional household per each region. The total income of the regional household is calculated as the sum of its labour income and its capital income:

$$Y_i = \left(LS_i - UNEMP_i\right) \cdot PW_i - LROW_i \cdot PLROW \cdot ER + \sum\nolimits_{Si} \left(K_{Si,i} \cdot RK_{Si,i}\right) \tag{1}$$

The labour income of the regional household is calculated as the total endowment of labour in the region minus the regional unemployment multiplied by the region-specific wage rate minus the net amount of regional labour supplied abroad multiplied by the foreign wage rate. It is assumed that the return to capital used by all sectors located in the region is allocated to the regional household. This way the capital income of the regional household is calculated as the sum over all regional sectors of their capital inputs multiplied by the sector-specific rate of return to capital and the exchange rate/terms of trade between the country and the rest of the world.

Besides the income from labour and capital each regional household also receives the unemployment benefits and other social transfers from the federal government. The money received by the regional household are spent on consumption of goods and services, transportation trips, savings and income taxes. The consumption budget of the household is the amount of money spent on the purchase of goods and services, which contribute to the household's utility. The higher the consumption of these services and goods is, the more utility the household receives. It is assumed that consumption of the transportation services does not directly contribute to the utility of the household.

The consumption budget of the regional household is calculated as its net income plus the social transfers from the government plus the unemployment benefits minus the households savings and spending on travel trips including commuting, education, shopping and other trips:

$$\begin{aligned}&CBUD_i = Y_i \cdot (1 - ty) + TRF_i \cdot GDPDEF + UNEMP_i \cdot PW_i \cdot trep_i - SH_i \\ &- \sum\nolimits_j \big(Tmoney_{i,j} \cdot LCM_{i,j} + SHOPTRIPS_{i,j} \cdot SHOPMONT_{i,j} \\ &+ OTHTRIPS_{i,j} \cdot OTHMONT_{i,j} + EDUTRIPS_{i,j} \cdot EDUMONT_{i,j}\big) \\ &\cdot \sum\nolimits_{Sj=transport} \big(P_{Sj,i} \cdot \big(1 - sc_{Sj} + tc_{Sj} + vatc_{Sj} + exst_{Sj}\big)\big)\end{aligned} \tag{2}$$

The savings of the regional household is calculated as a fixed proportion of its total disposable income that consists of the household's net income plus the social transfers from the government:

$$SH_i = mps_i \cdot \big(Y_i \cdot (1 - ty) + TRF_i \cdot GDPDEF + UNEMP_i \cdot PW_i \cdot trep_i\big) \tag{3}$$

## IV.2 Households' utility maximization problem

The amounts of the goods and services bought by the regional household are determined according to the utility-maximization problem, where the household maximizes the following LES-utility function:

$$U_i = \prod_{Si} (C_{Si,i} - \mu H_{Si,i})^{\alpha H_{Si,i}} \tag{4}$$

The utility from consumption is associated only with the amount of good and service which is higher than its subsistence consumption level. The regional household defines its consumption levels such as to maximize the LES utility function under the budget constraint that the total expenditures of the household are equal to its consumption budget. This utility problem is associated with the following solution for the optimal consumption levels:

$$\begin{aligned}&P_{Si,i} \cdot \big(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\big) \cdot C_{Si,i} = P_{Si,i} \cdot \big(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\big) \\ &\cdot \mu H_{Si,i} + \alpha H_{Si,i} \cdot \Big(CBUD_i - \sum\nolimits_{Sj} \big(\mu H_{Sj,i} \cdot P_{Sj,i} \cdot \big(1 - sc_{Sj} + tc_{Sj} + vatc_{Sj} + exst_{Sj}\big)\big)\Big)\end{aligned} \tag{5}$$

## IV.3 Households' welfare

The welfare of an individual regional household is calculated as the percentage share of the equivalent variation associated with a certain economic changes in the total income of the household:

$$EV_i = \left(\left(\frac{PEV_i^0}{PEV_i}\right) \cdot SII_i - SID_i^0\right) \cdot \frac{1}{Y_i} \cdot 100 \tag{6}$$

The calculation of the equivalent variation measure according to this formula is based on the unit price of an additional household's utility and on the level of the household's budget associated with the utility level. This budget does not include the spending necessary to pay for the subsistence levels of consumption. The subscript '0' refers to the initial baseline values of the utility price and the utility budget.

The price of an additional unit of utility obtained by the household is derived according to the following formula:

$$PEV_i = \prod\nolimits_{Si=CZSii} \Big(\big(P_{Si,i} \cdot \big(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\big)\big)^{\alpha H_{Si,i}}\Big) \tag{7}$$

The price of the unit of the household's utility depends upon the after-tax prices of goods and services as well as the utility shares. The household's budget associated with its utility level is calculated as the total household's consumption budget minus the spending of the households on the provision of its subsistent levels of consumption:

$$SII_i = CBUD_i - \sum\nolimits_{Si} \; (\mu H_{Si,i} \cdot P_{Si,i} \cdot (1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si})) \qquad (8)$$

## IV.4 Labour market and migration

The total endowment of labour in the region is calculated as the initial labour endowment plus the sum of the net inflow of labour from the rest of the regions minus the time spent on the commuting transportation trips:

$$\begin{aligned} LS_i = LS_i^0 + \sum_j \; (LMIG_{j,i} - LMIG_{i,j}) \\ - \sum\nolimits_j \left(Ttime_{i,j} \cdot LCM_{i,j} \;\right) \end{aligned} \qquad (9)$$

The migration flow from region i to region j is derived according to the discrete choice migration generation function. The decision to move to another region of the' country is split into two steps. At the first step, a household decides whether to move from the region or not. The choice to move is based upon the sector specific preferences and the difference between the average regional household's utility and the household's utility of this particular region. At the second step the household decides to which region it is going to move. This choice is based on the region specific preferences and the level of the household's utility in the destination region:

$$\begin{aligned} LMIG_{i,j} = \sum_k LS_k \cdot (Bmig_i + \sum_k \; U_k / N - U_i) / \sum_l \; (Bmig_l + \sum_k \; U_k / N - U_l) \\ \cdot (Amig_{i,j} + U_j) / \sum_k \; (Amig_{i,k} + U_k) \end{aligned} \qquad (10)$$

The market clearing condition for the labour market in each of the model's regions is written down as the equality between the sum of the labour demand of all the production sectors in the region and the total supply of labour in the region. The total regional labour supply is calculated as the sum of the commuting flows from all the regions in the country to the particular region under consideration:

$$\sum\nolimits_{Si} \left(L_{Si,i}\right) = \sum\nolimits_j \left(LCM_{i,j}\right) \qquad (11)$$

An average wage of the household in the region depends upon its commuting flows towards the regions in the rest of the country and the region-specific wage rates. The average household's wage is calculated as the weighted average of the wage rate in all the regions in the countries, where the proportions of labour from this region working in other regions of the country are used as the weights:

$$PW_i \cdot \left(LS_i - UNEMP_i\right) = \sum\nolimits_j \left(LCM_{j,i} \cdot PL_j\right) \qquad (12)$$

An unemployment level in the region is determined according to the wage curve which describes the relationship between the rate of voluntary unemployment in the region and other relevant variables including the level of the real regional wages. Increase in the level of the real wages leads to the decrease in the regional unemployment rate:

$$\log\left(\frac{PW_i}{INDEX_i}\right) = ParW0_i + ParW1 \cdot \log\left(\frac{\sum_{Si} XD_{Si,i}}{\sum_{Si} L_{Si,i}}\right) + ParW3 \cdot \log\left(1 - \frac{UNEMP_i}{LS_i}\right) \tag{13}$$

The implemented wage equation is based on the econometric estimates done by Guichard S and Laffargue J-P (2000) for a range of European countries.

The total unemployment level in the region is equal to the total labour endowment multiplied by the regional unemployment level:

$$UNEMP_i = LS_i \cdot UNRATE_i \tag{14}$$

## IV.5 Passenger transport and commuting

Mathematical specification used for the generation of the commuting trips between the regions differs from the generation of other passenger trips and is based on the formulation presented in the initial RAEM mathematical formulation. The initial RAEM formulation has been developed by Mark Thissen, Jos van Ommeren, Jan Oosterhaven, Piet Rietveld and Peter Zwaneveld in the beginning of the RAEM project.

The attractions for the generation of the commuting trips are the labour endowment in the region of origin and the total demand for labour at the region of destination. The generalized transport costs between the pair of regions negatively influence the number of the commuting trips between them. The labour commuting between each pair of regions is calculated according to the following formula:

$$\begin{aligned} LCM_{j,i} &= aM_{j,i} \cdot \left(LS_i - UNEMP_i\right)^{\alpha M_{j,i}} \cdot \left(\sum_{Si}\left(L_{Si,j}\right)\right)^{\left(1-\alpha M_{j,i}\right)} \\ &\cdot \exp\left(-\beta T_{j,i} \cdot \left(Ttime_{j,i} + Tmoney_{j,i}\right)\right) \end{aligned} \tag{15}$$

The generation of other passenger trips is based on a generation-distribution model which follows the structure of the constrained gravity model. The trips are generated based on the levels of the generation variables at the regions of destination. For all passenger trips (except commuting) we use the stock of labour in the region as the generation variable.

The choice of the attraction variables for the generation of the trips depends upon the type of the trip and is different for education, shopping and other passenger trips. In all the case we take total output of the corresponding service sector to be the major attraction variable. The total transport costs are calculated as the sum of the monetary and time costs associated with each pair of regions in the country. Increase in the transportation costs of a particular pair of the regions negatively influences the number of the trips between these pair. However, it does not have any effect upon the total number of the passenger trips generated at the region of origin.

The attractions for the education trips are the total outputs of the education services at the region of destination. The number of the education trips between each pair of regions is calculated according to the following formula:

$$\begin{aligned} EDUTRIPS_{i,j} &= \gamma EDU_i \cdot LS_i \\ &\cdot \frac{\beta EDU_{i,j} \cdot \sum_{Si=eduSi}\left(XD_{Si,j}\right) \cdot \exp\left(-\left(EDUTIME^0_{i,j} + EDUMONT^0_{i,j}\right)\right)}{\sum_k \left(\beta EDU_{i,k} \cdot \sum_{Si=eduSi}\left(XD_{Si,k}\right) \cdot \exp\left(-\left(EDUTIME^0_{i,k} + EDUMONT^0_{i,k}\right)\right)\right)} \end{aligned} \tag{16}$$

The attractions for the shopping trips are the total outputs of the retail industry at the region of destination. The number of the shopping trips between each pair of regions is calculated according to the following formula:

$$SHOPTRIPS_{i,j} = \gamma SHOP_i \cdot \frac{Y_i}{INDEX_i} \cdot \frac{\beta SHOP_{i,j} \cdot \sum_{Si=shopSi}\left(XD_{Si,j}\right) \cdot \exp\left(-\left(SHOPTIME^0_{i,j} + SHOPMONT^0_{i,j}\right)\right)}{\sum_k \left(\beta SHOP_{i,k} \cdot \sum_{Si=shopSi}\left(XD_{Si,k}\right) \cdot \exp\left(-\left(SHOPTIME^0_{i,k} + SHOPMONT^0_{i,k}\right)\right)\right)} \tag{17}$$

The attractions for the other/travel trips are the total outputs of the tourism and hotel industry at the region of destination. The number of the education trips between each pair of regions is calculated according to the following formula:

$$OTHTRIPS_{i,j} = \gamma OTHER_i \cdot \frac{Y_i}{INDEX_i} \cdot \frac{\beta OTHER_{i,j} \cdot \sum_{Si=travelSi}\left(XD_{Si,j}\right) \cdot \exp\left(-\left(OTHTIME^0_{i,j} + OTHMONT^0_{i,j}\right)\right)}{\sum_k \left(\beta OTHER_{i,k} \cdot \sum_{Si=travelSi}\left(XD_{Si,k}\right) \cdot \exp\left(-\left(OTHTIME^0_{i,k} + OTHMONT^0_{i,k}\right)\right)\right)} \tag{18}$$

## IV.6 Monopolistic competition

In each sector there operates a certain number of the firms that produce slightly differentiated goods and services. Given that there is no statistical data, which describes the production process of each firm in the industry, there is made an assumption that all firms are homogenous and have the same production technology and the same output size.

It is assumed that the production of the firms is characterised by the increasing returns and that they operate under the condition of the monopolistic competition. The fixed production costs of the firm are related to its initial establishment in the industry and include both labour and capital costs. Each new firm produces one particular type of the product type/variety. The firms charge prices higher then their marginal costs in order to be able to cover their fixed costs. Since consumers have widely differentiated preferences with respect to the types/varieties of goods and services produced by the firms, they purchase output of all the firms in the sector.

Given that the entry to all the industries is assumed to be free, the number of the monopolistic firms in each sector is determined by the condition that the total costs of the firms equal its total revenues. Once the firms in the industry start making profits several new firms enter the market and drive total profits down to zero again. The total number of the firms operating in a sector is defined thus by the zero profit condition for the sector as a whole:

$$\begin{aligned} & NF_{Si,i} \cdot elas\,\mathrm{Re}\,g_{Si,i} \cdot (fcL_{Si,i} + \frac{1}{NF_{Si,i}} \cdot \sum_j \; (BTRIPS_{Si,i,j} \cdot BTIME_{i,j}) \\ & + fcK_{Si,i}) \cdot INDEX_i = XD_{Si,i} \cdot PD_{Si,i} \end{aligned} \tag{19}$$

The price of the goods or services produced by a monopolistically competitive sector dependents negatively upon both the number of the operating firms and upon the elasticity of substitution between the varieties of a good or a service produced by the firms. Under the assumption that the firms operating in a sector are identical, the price of a monopolistically competitive sector is derived according to the following formula:

$$PDC_{Si,i} = PD_{Si,i} \cdot AUXV_{Si,i} \tag{20}$$

where

$$AUXV_{Si,i} = \left(NF_{Si,i}\right)^{\frac{1}{\left(1-elas\,\mathrm{Reg}_{Si,i}\right)}} \tag{21}$$

## IV.7 Firms' profit maximization problem

Regional sectors use labour, capital and intermediate goods in their production process. Labour and capital inputs of the sectors are combined according to the Constant Elasticity of Substitution (CES) technology, whereas the intermediate goods are used in the fixed proportions to the total output of the sectors according to the Leontief technology. The sectors define the amount of their inputs per unit of output based on the cost minimization principle. The resulting amounts of capital, labour and intermediate goods depend upon the production technology of the sectors and upon the prices of the sectoral inputs.

The value of the capital-labour CES bundle is derived as a fixed share of the total output of the sector. This bundle consists only of the variable capital and labour production costs:

$$KL_{Si,i} = ioKL_{Si,i} \cdot XD_{Si,i} \tag{22}$$

The composite price of the capital-labour bundle is derived as a weighted average of the after-tax prices of capital and labour:

$$\begin{aligned} PKL_{Si,i} \cdot KL_{Si,i} &= \left( K_{Si,i} - NF_{Si,i} \cdot fcK_{Si,i} \right) \cdot \left( \left(1 + tk_{Si}\right) \cdot RK_{Si,i} + \delta_{Si,i} PI \right) \\ &+ PL_i \cdot \left( L_{Si,i} - NF_{Si,i} \cdot fcL_{Si,i} + \sum_j \left( BTRIPS_{Si,i,j} \cdot BTIME_{Si,i,j} \right) \right) \\ &\cdot \left(1 + tl1_{Si} + \left(1 + tl1_{Si}\right) \cdot tl_{Si}\right) \end{aligned} \tag{23}$$

The total capital and labour costs of the sector are derived as the sum of the variables costs resulting form the costs minimization problem plus the total fixed costs of the sector calculated as the number of the operating firms multiplied by the fixed costs per firm:

$$\begin{aligned} K_{Si,i} &= KL_{Si,i} \left( \frac{\gamma K_{Si,i}}{\left(1 + tk_{Si}\right) \cdot RK_{Si,i} + \delta_{Si,i} \cdot PI} \right)^{\sigma KL_{Si,i}} \cdot \left(PKL\right)^{\sigma_{Si,i}} \\ &\cdot \left(aKL_{Si,i}\right)^{\left(\sigma KL_{Si,i} - 1\right)} + NF_{Si,i} \cdot fcK_{Si,i} \end{aligned} \tag{24}$$

The fixed labour costs of an individual firm include the time costs of the business trips. The idea behind this assumption is to relate the number of the business trips to the number of the operating firms and not to the total output of the sector. It is assumed that the more firms operate in the sector the more time is spent on business trips, such that the total time costs of the business trips per firm stays approximately the same.

$$\begin{aligned} L_{Si,i} &= KL_{Si,i} \left( \frac{\gamma L_{Si,i}}{PL_{Si} \cdot \left(1 + tl1_{Si} + \left(1 + tl1_{Si}\right) \cdot tl_{Si}\right)} \right)^{\sigma KL_{Si,i}} \cdot \left(PKL\right)^{\sigma_{Si,i}} \\ &\cdot \left(aKL_{Si,i}\right)^{\left(\sigma KL_{Si,i} - 1\right)} + NF_{Si,i} \cdot fcL_{Si,i} + \sum_j \left( BTRIPS_{Si,i,j} \cdot BTIME_{Si,i,j} \right) \end{aligned} \tag{25}$$

The total production costs of the sector is calculated as the sum of the capital costs, which include depreciation, labour costs, the costs of intermediate inputs and the monetary costs of the business trips:

$$\begin{aligned} & PD_{Si,i} \cdot XD_{Si,i} \cdot \left(1 - txd_{Si} + sp_{Si}\right) = K_{Si,i} \cdot \left(\left(1 + tk_{Si}\right) \cdot RK_{Si,i} + \delta_{Si,i} \cdot PI\right) \\ & + PL_i \cdot L_{Si,i} \cdot \left(1 + tl1_{Si} + \left(1 + tl1_{Si}\right) \cdot tl1_{Si}\right) + \sum\nolimits_{Sj} \left(io_{Sj,Si,i} \cdot XD_{Si,i} \cdot P_{Sj,i}\right) \\ & + \sum\nolimits_{j} \left(BTRIPS_{Si,i,j} \cdot BMONT_{i,j}\right) \cdot \sum\nolimits_{Sj=transport} \left(P_{Sj,i}\right) \end{aligned} \tag{26}$$

## IV.8 Business trips

The total number of the business trips of the sector is assumed to be linearly dependent upon the number of the operating firms:

$$BRTRIPST_{Si,i} = \beta BT_{Si,i} \cdot NF_{Si,i} \tag{27}$$

The total number of the business trips generated by the sector is distributed between the destinations in other regions of the country according to the gravity equation:

$$\begin{aligned} & BTRIPS_{Si,i,j} = BTRIPST_{Si,i} \\ & \cdot \frac{\alpha BT_{Si,i,j} \cdot BTSHARE_{i,j} \cdot \exp\left(-\left(BMONT_{i,j} + BTIME_{i,j}\right)\right)}{\sum\nolimits_{k} \alpha BT_{Si,i,k} \cdot BTSHARE_{i,k} \cdot \exp\left(-\left(BMONT_{i,k} + BTIME_{i,k}\right)\right)} \end{aligned} \tag{28}$$

The number of the business trips from region i to region j depends positively upon the share of the trade with the region j in the total trade of region i. The number of the trips depends negatively upon the generalised transport costs between the regions.

The share of the trade with the region j in the total trade of region i is calculated according to the following formula:

$$BTSHARE_{j,i} = \frac{\sum\nolimits_{Sj} \left(XDDE_{Sj,j,i} + XDDE_{Sj,i,j}\right)}{\sum\nolimits_{Sj} \sum\nolimits_{k} \left(XDDE_{Sj,j,k} + XDDE_{Sj,k,j}\right)} \tag{29}$$

## IV.9 Interregional trade

The formulation of an interregional trade part of the model is based on the assumption of heterogeneity between the goods and services produced in different domestic regions. The substitution possibilities between the commodities produced in different regions are described by the CES production function, according to which the regional services and goods are used in a certain proportion in order to produce a composite domestic commodity.

The composite domestic commodity consists of the goods and services produced in all the regions of the country. The demand for the regional commodities is calculated according to the following formula:

$$XDDE_{Si,j,i} = XDD_{Si,i} \cdot \left( \frac{\gamma A4_{Si,j,i}}{PDC_{Si,j} + PTM \cdot trmV_{Si,j,i}} \right)^{\sigma A1_{Si,i}} \cdot \left(PDDT_{Si,i}\right)^{\sigma A1_{Si,i}} \cdot \left(aA_{Si,i}\right)^{\left(\sigma A1_{Si,i} - 1\right)} \tag{30}$$

The price of the composite domestic goods and services is derived as the weighted average of the prices of the commodities bought from all domestic regions:

$$PDDT_{Si,i} \cdot XDD_{Si,i} = \sum_{j} \left( XDDE_{Si,j,i} \cdot (PDC_{Si,i} + PTM \cdot trm_{j,i,Si}) \right) \tag{31}$$

Under the assumption of the monopolistic competition combined with the use of the Dixit-Stiglitz varieties approach the utility of the consumption of a certain commodity depends positively upon the number of the available varieties of this commodity. In the model it is assumed that each firm inside of the sector produces only one type/variety of the commodity. Hence, the number of the varieties available to the consumers is equal to the number of the operating firms. An increase in the utility of the consumers resulting from an increase in the number of the varieties is captured via an increase in the total quantity of the commodity produced by the sector as follows:

$$XD_{Si,i} = \left( \sum_{j} \left( XDDE_{Si,i,j} \right) + EEU25_{Si,i} + EROW_{Si,i} \right) \cdot AUXV_{Si,i} \tag{32}$$

## IV.10 External trade

The formulation of an international trade part of the model is based on the Armington assumption of heterogeneity between the goods and services produced abroad and domestically. Goods and services produced aboard cannot be perfectly substituted with the domestically produced ones. The substitution possibilities between domestic and foreign commodities are described by the CES production function, according to which domestic and foreign commodities are used in a certain proportion in order to produce a composite commodity used in consumption by the domestic firms and households.

The composite commodity used by the domestic firms and household consists of the goods and services produced domestically and imported either from the EU25 countries or from the rest of the world. The demand for the domestically produced goods is calculated according to the following formula:

$$XDD_{Si,i} = X_{Si,i} \cdot \left( \frac{\gamma A3_{Si,i}}{PDDT_{Si,i}} \right)^{\sigma A_{Si,i}} \cdot P_{Si,i}^{\ \sigma A_{Si,i}} \cdot aA_{Si,i}^{\ \sigma A_{Si,i}-1} \tag{33}$$

The demand for the commodities imported from EU25 countries is calculated according to the following formula:

$$MEU25_{Si,i} = X_{Si,i} \cdot \left( \frac{\gamma A1_{Si,i}}{PMEU25_{Si}} \right)^{\sigma A_{Si,i}} \cdot \left( P_{Si,i} \right)^{\sigma A_{Si,i}} \cdot \left( aA_{Si,i} \right)^{(\sigma A_{Si,i}-1)} \tag{34}$$

The demand for the commodities imported from other foreign countries is calculated according to the following formula:

$$MROW_{Si,i} = X_{Si,i} \cdot \left( \frac{\gamma A2_{Si,i}}{PMROW_{Si}} \right)^{\sigma A_{Si,i}} \cdot \left( P_{Si,i} \right)^{\sigma A_{Si,i}} \cdot \left( aA_{Si,i} \right)^{(\sigma A_{Si,i}-1)} \tag{35}$$

The price of the composite goods and services consumed in the country is derived as the weighted average of the prices of the goods and services produced domestically and imported form abroad:

$$\begin{aligned} P_{Si,i}.X_{Si,i} &= \sum_{j} \left( XDDE_{Si,j,i} \cdot \left( PDC_{Si,j} + PTM \cdot trmV_{Si,j,i} \right) \right) \\ &+ PMEU25_{Si} \cdot MEU25_{Si,i} + PMROW_{Si} \cdot MROW_{Si,i} \end{aligned} \tag{36}$$

The prices of the commodities imported to the country from EU25 countries and from the rest of the world in foreign currency are exogenously fixed in the model and their prices in the domestic currency are calculated according to the following formulas, where the subscript '0' refers to the commodity prices in foreign currency:

$$PMROW_i = PWMROW_i^0 \cdot ER \tag{37}$$

$$PMEU25_i = PWMEU25_i^0 \cdot ER \tag{38}$$

Domestic sectors have the possibility to export their production to the EU25 countries and to the rest of the world. Each sector decides upon the amount of commodities to be produced for export and their decisions depend upon their unit production costs, the level of the prices in the domestic economy and the price elasticity. Increase in the production costs of the sector relative to the average level of the prices in the economy leads to the decrease in the production activity of the sector and hence reduces the level of its exports.

The levels of the exports of each regional domestic sector to the EU25 countries and to the rest of the world are derived according to the following formulas:

$$EROW_{Si,i} = EROW_{Si,i}^0 \cdot \left( \frac{INDEX_i}{PD_{Si,i}} \right)^{elasE_{Si}} \tag{39}$$

$$EEU25_{Si,i} = EEU25_{Si,i}^0 \cdot \left( \frac{INDEX_i}{PD_{Si,i}} \right)^{elasE_{Si}} \tag{40}$$

## IV.11 Investment sector

The total domestic savings consists of the savings made by all regional households, government and the regional sectors. The savings of the regional sectors are assumed to be equal to their depreciation costs. The total domestic savings are calculated according to the following formula:

$$S = \sum_i (SH_i) + SG \cdot GDPDEF + \sum_{Si} \sum_i (delta_{Si,i} \cdot K_{Si,i} \cdot PI) \tag{41}$$

The total investments in the economy consist of domestic savings, the savings/investments received from the EU25 countries and from the rest of the world minus the total changes in stocks:

$$IT = S + SEU25 \cdot ER + SROW \cdot ER - \sum_{Si} \sum_i (SV_{Si,i} \cdot P_{Si,i}) \tag{42}$$

The total investments are spent on buying physical investments goods from various domestic regions, where the demand for them is determined according to the Cobb-Douglas demand function:

$$I_{Si,i} \cdot P_{Si,i} \cdot (1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}) = \alpha I_{Si,i} \cdot IT \tag{43}$$

The nominal rate of return in the economy is calculated as the average return to capital of all domestic sectors:

$$RGD_i = \frac{\sum_{Si} (RK_{Si,i} \cdot K_{Si,i})}{\sum_{Sj} (K_{Sj,i})} \tag{44}$$

The price of additional unit of the composite physical investment good is calculated in accordance to the Cobb-Douglas demand function and has the following form:

$$PI = \prod_{Si} \prod_{i} \left( \frac{P_{Si,i} \cdot \left(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\right)}{\alpha I_{Si,i}} \right)^{\alpha I_{Si,i}} \tag{45}$$

## IV.12 Governmental sector

The federal government in the economy is responsible for the collection of the taxes from all economic agents, paying subsidies and making transfers. It also spends part of its budget on buying different commodities used to provide particular services to the population.

The total tax revenues of the federal government are calculated as the sum of the social security taxes paid by the employers and employees, profit taxes of the firms, taxes on production and consumption, VAT taxes and excises:

$$\begin{aligned} TAXR = {} & \sum\nolimits_i \Big( \sum\nolimits_{Si=XDZSii} \big( PL_i \cdot L_{Si,i} \cdot \left( tl1_i + \left(1 + tl1_i\right) \cdot tl1_i + tk_{Si} \cdot K_{Si,i} \cdot RK_{Si,i} \right. \\ & \left. + txd_{Si} \cdot XD_{Si,i} \cdot PD_{Si,i} \right) \\ & + \sum\nolimits_{Si \neq transport} \left( \left( tc_{Si} + vatc_{Si} + exst_{Si} \right) \cdot P_{Si,i} \cdot \left( C_{Si,i} + I_{Si,i} + CG_{Si,i} \right) + ty \cdot Y_i \right) \\ & + \sum\nolimits_{Si=transport} \Big( \left( tc_{Si} + vatc_{Si} + exst_{Si} \right) \cdot P_{Si,i} \cdot \Big( C_{Si,i} + \sum\nolimits_j \big( Tmoney_{i,j} \\ & \cdot LCM_{i,j} + SHOPTRIPS_{i,j} \cdot SHOPMONT_{i,j} + OTHTRIPS_{i,j} \cdot OTHMONT_{i,j} \\ & + EDUTRIPS_{i,j} \cdot EDUMONT_{i,j} \big) + I_{Si,i} + CG_{Si,i} \Big) \Big) \end{aligned} \tag{46}$$

The total subsidies of the federal government consist of the subsidies on production and consumption:

$$\begin{aligned} SUBS = {} & \sum\nolimits_i \Big( \sum\nolimits_{Si} \left( sp_{Si} \cdot XD_{Si,i} \cdot PD_{Si,i} \right) + \sum\nolimits_{Si \neq transport} \left( sc_{Si} \cdot P_{Si,i} \cdot \left( C_{Si,i} + I_{Si,i} + CG_{Si,i} \right) \right) \Big) \\ & + \sum\nolimits_{Si=transport} \ (sc_{Si,i} \cdot P_{Si,i} \cdot (C_{Si,i} + \sum\nolimits_j \ (Tmoney_{i,j} \cdot LCM_{i,j} + SHOPTRIPS_{i,j} \\ & \cdot SHOPMONT_{i,j} + OTHERTRIPS_{i,j} \cdot OTHMONT_{i,j} + EDUTRIPS_{i,j} \\ & \cdot EDUMONT_{i,j})) \end{aligned} \tag{47}$$

The consumption budget of the federal government spent on the purchase of various commodities consists of the total tax revenues minus total subsidies minus the unemployment benefits, households' transfers and governmental savings plus the transfers to the federal government form abroad. The consumption budget of the federal government is distributed between the consumption of various regional commodities according to the Cobb-Douglas demand function:

$$\begin{aligned} & CG_{Si,i} \cdot P_{Si,i} \cdot \left(1 - sc_i + tc_i + vatc_i + exst_i\right) = \alpha G_{Si,i} \cdot \big( TAXR - SUBS \\ & - \sum\nolimits_j \left( TRF_j \cdot GDPDEF + UNEMP_j \cdot PW_j \cdot trep_j \right) - SG \cdot GDPDEF + TREU25 \cdot ER \big) \end{aligned} \tag{48}$$

## IV.13 Market equilibrium conditions

Markets for goods and services are in equilibrium in each region of the country. According to the market clearing condition the total supply of a certain commodity in region i is equal to the sum of the demand of the regional household, region-specific demand of the federal government, region-specific demand for physical investment goods, changes in stocks, region-specific demand for commodities used for production of freight trade and transport margins, intermediate demands of the regional production sectors. For the transport services sector, the demands of firms and households for passenger transport need to be added to the market clearing condition:

$$X_{Si,i} = C_{Si,i} + CG_{Si,i} + I_{Si,i} + SV_{Si,i} + TMX_{Si,i} + \sum_{Sj} (io_{Si,Sj,i} \cdot XD_{Sj,i}) \quad \text{for} \quad Si \neq \textit{transport} \tag{49}$$

In order to write the equation for the demand for transport services we nee to add an additional term related to the total amount of the passenger trips to equation (49):

$$\begin{aligned} X_{Si,i} &= C_{Si,i} + CG_{Si,i} + I_{Si,i} + SV_{Si,i} + TMX_{Si,i} + \sum_{Sj} (io_{Si,Sj,i} \cdot XD_{Sj,i}) \\ &+ \sum_{Sj} \sum_{j} (BTRIPS_{Sj,i,j} \cdot BMONT_{i,j}) + \sum_{j} (Tmoney_{i,j} \cdot LCM_{i,j} \\ &+ SHOPTRIPS_{i,j} \cdot SHOPMONT_{i,j} + OTHTRIPS_{i,j} \cdot OTHMONT_{i,j} \\ &+ EDUTRIPS_{i,j} \cdot EDUMONT_{i,j}) \end{aligned} \quad \text{for } Si = \textit{transport} \tag{50}$$

## IV.14 Freight trade and transport services, changes in stocks

Nationwide freight transport and trade services are a combination of the transportation and trade produced by the respective sectors in different regions of the country. The total demand for the freight transport and trade services in the country is calculated as the sum over all bi-regional trade flows multiplied by the commodity specific transportation margin. The level of this margin depends upon the regions of origin and destination and increases with distance. The total demand for the freight transport and trade services are split between different regions according to the fixed proportion consistent with the initial data:

$$TMX_{Si,i} = atm_{Si,i} \cdot \sum_{Sj} \sum_{k} \sum_{j} \left( trm_{Sj,k,j} \cdot XDDE_{Sj,k,j} \right) \tag{51}$$

The unit price of nationwide freight transport and trade services is calculated as the weighted average of the prices of the services (transport and trade) necessary for its provision:

$$PTM = \sum_{Si} \sum_{i} \left( atm_{Si,i} \cdot P_{Si,i} \right) \tag{52}$$

Region-specific changes in stocks are calculated as a fixed proportion of the regional demand for the commodities:

$$SV_{Si,i} = svs_{Si,i} \cdot X_{Si,i} \tag{53}$$

## IV.15 Economic indicators

The total real imports of the county are calculated as the sum of the values of the imports from the EU25 countries and from the rest of the world divided by the import price index:

$$MT = \frac{1}{INDEXM} \sum_{Si} \sum_{i} \left( PMEU25_{Si} \cdot MEU25_{Si,i} + PMROW_{Si} \cdot MROW_{Si,i} \right) \tag{54}$$

The total real exports of the country are calculated as the sum of the values of the exports to the EU25 countries and to the rest of the world divided by the exports price index:

$$ET = \frac{1}{INDEXE} \sum_{Si} \sum_{i} \left( EEU25_{Si,i} \cdot PDC_{Si,i} + EROW_{Si,i} \cdot PDC_{Si,i} \right) \tag{55}$$

The exports price index is calculated as the ratio between the exports valued in present prices and the exports valued in the base-year prices:

$$INDEXE = \frac{\sum_{Si} \sum_{i} \left( EEU\,25_{Si,i} \cdot PDC_{Si,i} + EROW_{Si,i} \cdot PDC_{Si,i} \right)}{\sum_{Si} \sum_{i} \left( EEU\,25_{Si,i} \cdot PDC^{0}_{Si,i} + EROW_{Si,i} \cdot PDC^{0}_{Si,i} \right)} \tag{56}$$

The imports price index is calculated as the ratio between the imports valued in present prices and the imports valued in the base-year prices:

$$INDEXM = \frac{\sum_{Si} \sum_{i} \left( PMEU25_{Si} \cdot MEU25^{0}_{Si,i} + PMROW_{Si} \cdot MROW^{0}_{Si,i} \right)}{\sum_{Si} \sum_{i} \left( PMEU25^{0}_{Si} \cdot MEU25^{0}_{Si,i} + PMROW^{0}_{Si} \cdot MROW^{0}_{Si,i} \right)} \tag{57}$$

The regional consumer price index is calculated as the ratio between the base-year values of household's commodity consumption valued in the present prices and the base-year values of household's commodity consumption valued in the base-year prices:

$$INDEX_{i} = \frac{\sum_{Si=CZSii} C^{0}_{Si,i} \cdot P_{Si,i} \cdot \left(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\right)}{\sum_{Si=CZSii} C^{0}_{Si,i} \cdot P^{0}_{si,i} \cdot \left(1 - sc^{0}_{Si} + tc^{0}_{Si} + vatc^{0}_{Si} + exst^{0}_{Si}\right)} \tag{58}$$

The real GDP of the country is calculated as the sum of the total production minus intermediate inputs plus the households' consumption valued in the base-year prices:

$$\begin{aligned}
GDP = {} & \sum_{Si} \sum_{i} \left( XD_{Si,i} \cdot PD^{0}_{Si,i} \right) - \sum_{Sj} \sum_{Si} \sum_{i} \left( io_{Sj,Si,i} \cdot XD_{Si,i} \cdot P^{0}_{Sj} \right) \\
& - \sum_{j} \sum_{i} \sum_{Si} \left( BTRIPS_{Si,j,i} \cdot BMONT_{j,i} \cdot \sum_{Sj=transport} \left( P^{0}_{Sj,i} \right) \right) \\
& + \sum_{Si \neq transport} \sum_{i} \left( \left( tc_{Si} + vatc_{Si} + exst_{Si} - sc_{Si} \right) \cdot P^{0}_{Si,i} \cdot \left( C_{Si,i} + I_{Si,i} + CG_{Si,i} \right) \right) \\
& + \sum_{Si=transport} \sum_{i} \left( \left( tc_{Si} + vatc_{Si} + exst_{Si} - sc_{Si} \right) \cdot P^{0}_{Si,i} \cdot \left( C_{Si,i} + \sum_{j} \left( Tmoney_{i,j} \right. \right. \right. \\
& \cdot LCM_{i,j} + SHOPTRIPS_{i,j} \cdot SHOPMONT_{i,j} + OTHTRIPS_{i,j} \cdot OTHMONT_{i,j} \\
& \left. \left. \left. + EDUTRIPS_{i,j} \cdot EDUMONT_{i,j} \right) + I_{Si,i} + CG_{Si,i} \right) \right)
\end{aligned} \tag{59}$$

The nominal GDP of the country is calculated as the sum of the total production minus intermediate inputs plus the households' consumption valued in the present prices:

$$
\begin{aligned}
GDPC = {} & \sum_{Si}\sum_{i}(XD_{Si,i}\cdot P_{Si,i}) - \sum_{Sj}\sum_{Si}\sum_{i}(io_{Sj,Si,i}\cdot XD_{Si,i}\cdot P_{Sj,i}) \\
& - \sum_{j}\sum_{i}\sum_{Si}(BTRIPS_{Si,j,i}\cdot BMONT_{j,i}\cdot \sum_{Sj=transport}(P_{Sj,i})) \\
& + \sum_{Si\neq transport}\sum_{i}((tc_{Si}+vatc_{Si}+exst_{Si}-sc_{Si})\cdot P_{Si,i}\cdot (C_{Si,i}+I_{Si,i}+CG_{Si,i})) \\
& + \sum_{Si=transport}\sum_{i}((tc_{Si}+vatc_{Si}+exst_{Si}-sc_{Si})\cdot P_{Si,i}\cdot (C_{Si,i}+\sum_{j}(Tmoney_{i,j} \\
& \cdot LCM_{i,j}+SHOPTRIPS_{i,j}\cdot SHOPMONT_{i,j}+OTHTRIPS_{i,j}\cdot OTHMONT_{i,j} \\
& + EDUTRIPS_{i,j}\cdot EDUMONT_{i,j})+I_{Si,i}+CG_{Si,i}))
\end{aligned}
\tag{60}
$$

## *IV.16 Recursive dynamics*[1]

RAEM 3.0 has a recursive dynamic structure composed of a sequence of several temporary equilibria. The first equilibrium in the sequence is given by the benchmark year 2000. In each time period, the model is solved for an equilibrium given the exogenous conditions assumed for that particular period. The equilibria are connected to each other through capital accumulation. In the benchmark case, we assume that the economy is on a steady-state growth path, where all the quantity variables grow at the same rate and all relative prices remain unchanged. When a policy measure is implemented the economy enters on a transition path, until, after some time it has reached a new steady-state growth path (Ballard, Fullerton, Shoven and Walley, 1985). We are of course interested in the transition path induced by the policy measure and the characteristics of the new growth path.

The endogenous determination of investment behaviour is essential for the dynamic part of the model. Investment and capital accumulation in year t depend on expected rates of return for year t+1, which are determined by actual returns on capital in year t. This approach involves adaptive expectations. Thus, investment is not only a demand category in the model. In the dynamic economic processes a homogenous composite investment commodity is allocated between sectors according to the actual (year t) returns on capital in sector Si.

The equilibrium expected rate of return $RK_{Si,i,t}$ by sector and region in year t, is specified as an inverse logistic function (see Figure 1) of the proportionate growth in sector's capital stock (Dixon and Rimmer, 2002):

$$
\begin{aligned}
RK_{Si,i,t} = {} & RK^{0}_{Si,i,t} + (1/B_{Si,i})\cdot[(\ln(Kg_{Si,i,t} - Kg\min_{Si,i})) \\
& - \ln(Kg\max_{Si,i} - Kg_{Si,i,t}) - \ln(Ktrend_{Si,i} - Kg\min_{Si,i}) \\
& + \ln(Kg\max_{Si,i} - Ktrend_{Si,i})]
\end{aligned}
\tag{61}
$$

where $RK^{0}_{Si,i,t}$ is the sector's historically normal rate of return, $Kg_{Si,i,t}$ is the actual capital growth rate in the sector, $Kg\min_{Si,i}$ and $Kg\max_{Si,i}$ are the minimum and the maximum possible growth rates of capital in the sector, $Ktrend_{Si,i}$ is the sector's historically normal growth rate and $B_{Si,i}$ is a positive parameter. The minimum possible growth rate is set at the negative of the rate of

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[1] The description in this paragraph is based on Mohora (2006)

depreciation in the sector, while the maximum rate is set at $Ktrend_{Si,i}$ plus 0.062 in order to avoid unrealistically large simulated growth rate (Dixon and Rimmer, 2002).

Parameter $B_{Si,i}$ reflects the sensitivity of capital growth in sector sec to variations in its equilibrium expected rate of return. It is derived by differentiating the above equation with respect to $Kg_{Si,i,t}$ :

$$B_{Si,i} = SEA \cdot \left( \frac{Kg\max_{Si,i} - Kg\min_{Si,i}}{(Kg\max_{Si,i} - Ktrend_{Si,i}) \cdot (Ktrend_{Si,i} - Kg\min_{Si,i})} \right) \tag{62}$$

where:

$$SEA = \left( \frac{\partial RK_{Si,i,t}}{\partial Kg_{Si,i,t}} \right)^{-1} \tag{63}$$

The present value of purchasing a unit of capital to be used in the sector, in year t is defined as:

$$PVK_{Si,i,t} = -PI_t + [RRK_{Si,i,t+1} + PI_{t+1} \cdot (1 - \delta_{Si,i})] / [1 + RINT_t] \tag{64}$$

where $PI_t$ is the cost of buying a unit of capital (the price of composite investment commodity) in year t, $RRK_{Si,i,t}$ is the rental rate on sector's sec capital, $\delta_{Si,i}$ is the depreciation rate of the sector and $RINT_t$ is the interest rate in year t (Dixon and Rimmer, 2002). The purchase of one unit of capital in year t by sector involves an immediate expenditure, followed by two benefits in year t+1 which are discounted by $(1 + RINT_t)$: the rental value of an extra unit of capital in year t+1 and the value at which the depreciated unit of capital can be sold in year t+1. The actual rate of return on capital in sector sec in year t is further given by dividing both sides of by $PI_t$, and reflects the present value of an investment of one euro.

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2 The specification of the maximum possible growth rate implies that if the historically normal rate in a sector is 4 per cent, the upper limit in any year t would not exceed 10 per cent.

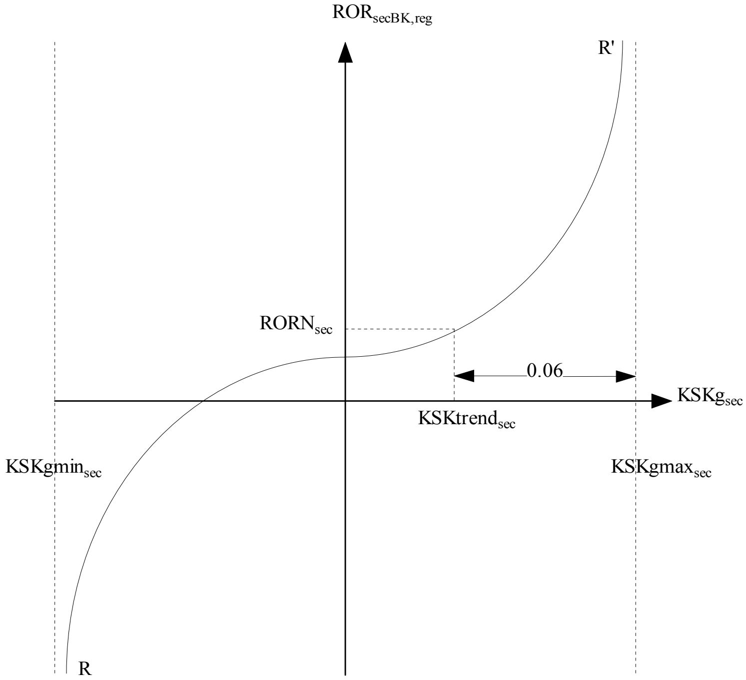

**Figure 1:** : The equilibrium expected rate of return for the sector (taken from Mohora (2006))

The expected rate of return $ROR_{Si,i,t}$ under adaptive expectations is derived as:

$$ROR_{Si,i,t} = -1 + [RK_{Si,i,t} / PI_t + (1-\delta_{Si,i})]/[1 + RINT_t / GDPDEF_t] \qquad (65)$$

where we assume that investors expect no change in the price of composite investment commodity and rental rates. The real rate of return in year t is given by:

$$RGD_{i,t} = \frac{\sum_{Si}\left(RK_{Si,i,t} \cdot K_{Si,i,t}\right)}{\sum_{Sj}\left(K_{Sj,i,t}\right)} \qquad (66)$$

and $[1 + RINT_t / GDPDEF_t]$ reflects the adaptive expectation of the real interest rate.

The capital stock of the sector in the next period (year t+1) is given by:

$$K_{Si,i,t+1} = (1-\delta_{Si,i}) \cdot K_{Si,i,t} + INV_{Si,i,t} \qquad (67)$$

The actual capital growth rate in sector sec can be derived from the above equation as:

$$\begin{aligned} Kg_{Si,i,t} &= [\alpha RK_{Si,i,t} \cdot Kg\max_{Si,i} \cdot (Ktrend_{Si,i} - Kg\min_{Si,i}) \\ &+ Kg\min_{Si,i} \cdot (Kg\max_{Si,i} - Ktrend_{Si,i})]/ \\ &[\alpha RK_{Si,i,t} \cdot (Ktrend_{Si,i} - Kg\min_{Si,i}) + (Kg\max_{Si,i} - Ktrend_{Si,i})] \end{aligned} \qquad (68)$$

where:

$$\alpha RK_{Si,i,t} = e^{B_{Si,i} \cdot (RK_{Si,i,t} - RK^{0}_{Si,i,t})} \qquad (69)$$

and the capital growth rate in terms of capital stock in year t+1 and the capital stock in year t is given by:

$$Kg_{Si,i,t} = K_{Si,i,t+1} / K_{Si,i,t} - 1 \tag{69}$$

From the previous equations we derive the investment carried out in the sector in year t:

$$\begin{aligned} INV_{Si,i,t} &= K_{Si,i,t} \cdot ([\alpha RK_{Si,i,t} \cdot Kg\max_{Si,i} \cdot (Ktrend_{Si,i} - Kg\min_{Si,i}) \\ &+ Kg\min_{Si,i} \cdot (Kg\max_{Si,i} - Ktrend_{Si,i})] / \\ &[\alpha RK_{Si,i,t} \cdot (Ktrend_{Si,i} - Kg\min_{Si,i}) + (Kg\max_{Si,i} - Ktrend_{Si,i})] + 1) \\ &- K_{Si,i,t} \cdot (1 - delta_{Si,i}) \end{aligned} \tag{70}$$

The model is solved dynamically with annual steps. The simulation horizon of the model has been set at 20 years but it can easily be extended. In between periods, some other variables like the transfers between firms, government and the rest of the world, and the balance of payments balance (foreign savings) are updated exogenously.

# V Model development

## V.1 Model overview

For the implementation of the RAEM 3.0 model a comprehensive social-economic database at the level of the NUTS3 Dutch regions has been constructed. The model database consists of the following data for the year 2000:

- National Social Accounting Matrix (SAM)
- Bi-regional input-output tables at the NUTS3 level (40 Dutch regions)
- Data on regional production, value-added and labour compensation by sector
- Transport data including the data on the passenger trips by type, their time and monetary costs
- Origin-Destination (OD) commuting matrix
- OD migration matrix
- Data on the interregional trade flows
- Regional unemployment data
- International trade statistics

The nonlinear programming method described in Canning and Wang (2005) is used in order to construct the regional level Social Accounting matrices for all the NUTS3 Dutch regions that are consistent with the national level Social Accounting Matrix.

The RAEM 3.0 model is implemented using the GAMS modelling language. It is formulated as a Mixed Complementarity Problem (MCP) and is solved with the help of the PATH solver. The model database is stored in the excel file which is directly read by GAMS. The model outputs are written to the specially designed output excel file, which allows the model users to easily access and analyze the results of the RAEM 3.0 model.

## V.2 Model database

### V.2.1 Social Accounting Matrix for the Netherlands[3]

A SAM is a square matrix in which each transaction is recorded only once in a cell of its own – it is conventionally agreed that the entries made in rows represent incomes or receipts, whilst the entries made in columns represent outlays or expenditures - so, for each row there is a corresponding column, i.e. for every income there exists a corresponding expenditure, with their totals being equal.

These figures will include both production and institutional accounts, which are subdivided into yet other accounts, defined in accordance with the goal of the study and the available information.

Thus, the SAM consists of a set of interrelated subsystems that, on the one hand, give an analytical picture of the studied economy in a particular accounting period and, on the other hand, serve as an instrument for assessing the effects of changes on the particular flows represented by it (injections and leakages in the system), which might be the result of policy measures. Therefore, the SAM can be seen as a working instrument for quantifying the flows in the economic circuit and for simulating the

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[3] General description of SAM is based on Mohora (2006)

effects resulting from any changes in such flows.

A SAM usually encompasses a somewhat less detailed supply and use table or input-output (IO) table. A clear distinction must be made between the IO table and the SAM. The essence of the IO table is the way industries are interrelated through transactions, while the SAM also presents the transactions and the transfers between the different types of economic agents like households, various categories of companies, government and the rest of the world.

In a simplified manner, the SAM is the transformation of the circular flow (see Figure 2) into a matrix of transactions between the various economic agents.

The households supply labour and capital to the firms (1), who organize the production activities. In return they receive payments for the use of their labour and capital factors (2). These payments may take the form of wages, interests or dividends. Furthermore, the households spend their income on products (3), which are delivered by firms/activities (4). The government is involved in transfers to and from households (5) and firms/activities (6). The transfers may refer to taxes on products and on production, subsidies, income taxes, social security contributions, social benefits, etc. The economic relations between the country and the rest of the world (ROW) are given by (7) in Figure 2. These relations may take the form of imports and exports of goods and services, compensation of employees and property income to and from ROW, taxes less subsidies on production and current transfers to and from ROW, adjustments for the change in net equity of households on pension funds reserves to and from ROW and capital transfers to and from ROW.

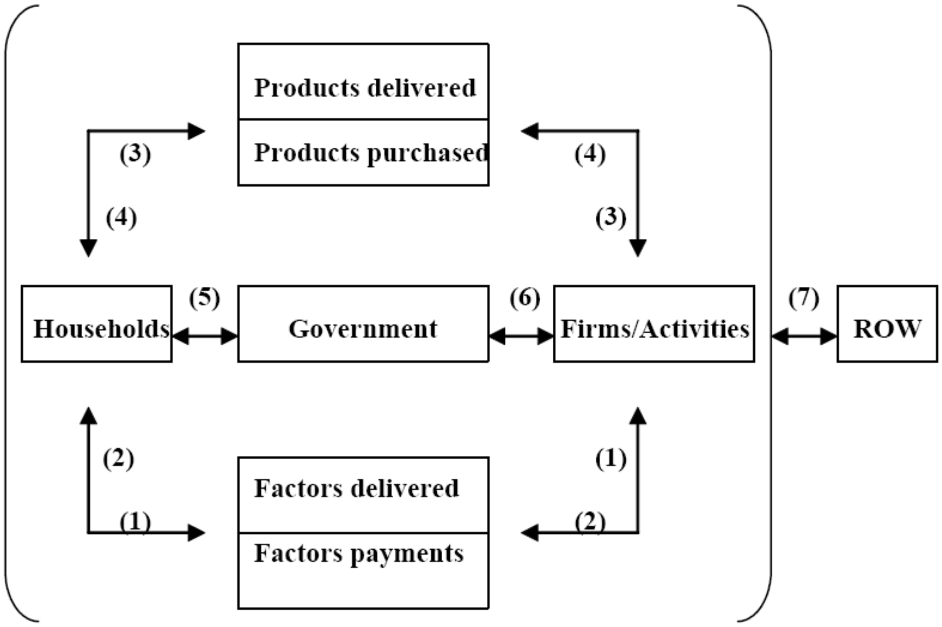

**Figure 2**: The circular flow of transactions between various economic agents.

A SAM is presented as a square matrix designed to provide a record of these transactions between various economic agents, using a single-entry form of booking. It can be represented as $T=[t_{jk}]$ , where j is the number of the row and k the number of the column. By convention, receipts are shown in the rows and outlays are shown in the columns. According to this convention, $t_{jk}$ is the value of all receipts of j from k during the accounting period. Correspondingly, $t_{kj}$ measures payments to k by j.

The design and the construction methods of the Social Accounting Matrices are not standardized but they should fulfil simultaneously two conditions:

- SAMs should be square matrices, in the sense that each account has its own row and column;
- the row totals and column totals for each account should be equal. This restriction can be written as: $T \cdot i = y = i' \cdot T'$ $i' = [1..........1]$ where $y$ is the column vector of row totals and column totals of $T$ and $T'$ is the transpose of the matrix $T$.

For the construction of the national-level SAM for the RAEM 3.0 model, we have used the supply and use tables for 2006 provided by Statistics Netherlands in combination with the national accounts data for 2006, which is also provided by Statistics Netherlands.

Several problems have been encountered and successfully solved, while working with those data. One of the serious problems is that exports and imports of supply and use tables from Statistics Netherlands include both own export and import as well as re-export. Inclusion of re-exports into supply and use tables leads to the situation where the total domestic output of the country becomes smaller then the country's export for several groups of products. In order to fix this problem we have used the study on re-export by TNO done in 2002. Results of this study have allowed us to split the total exports in the use table into re-exports and own exports. Further, only own exports figures were used for the construction of the SAM.

Commodity and sector classification used in the RAEM model differs strongly from the commodity and sector classification in supply and use tables of Statistics Netherlands. First, we have constructed the SAM based on the commodity and sector classification of the Statistics Netherlands. This SAM was balanced and checked for consistency. Second, the resulting detailed SAM was aggregated to match the sectoral classification of the RAEM model.

Detailed tax data by type is provided by Statistics Netherlands only on the national level as a part of the national accounts. The RAEM 3.0 model includes the representation of the following production and consumption related taxes:

- employer's social contribution
- employee's social contribution
- personal income tax
- corporate income tax
- value added tax
- excise tax
- other tax on production
- other tax on consumption

The national totals of tax receipts were split between the RAEM sectors and commodities proportionally to the SAM values they are associated with. These values include the total sectoral wages, total sectoral production etc. and depend upon the type of tax.

## V.2.2 Regional data for the Netherlands

The basic regional economic data used to calibrate model parameters are taken from the so called regional economic annual (REJ) data from the CBS Netherlands and are available for the year 2002.

In this source, the following variables are given for 37 sectors:

- Gross production
- Intermediairy use
- Value added
- Wages for employees
- Taxes not product related
- Subsidies not product related
- Gross exploitation surplus
- Labour (full time equivalent)

The data are consistent with the annual National Accounts of the Netherlands. The data for the REJ are processed and have been treated as follows:

- Estimation of empty (secret) cells
- Revision break double years (2001)
- Results aggregated to the 14 RAEM sectors

Especially the estimation of the empty cells requires explanation. It is done by a RAS procedure. Besides the empty cells one knows the rest values on the column (the region) and the row (the sector national). This basic information is in principle enough to perform a RAS procedure. But, Statistics Netherlands has used a specific method to cover information on individual firms to make the outcome of such a naïve use of a RAS procedure without any value. This method is that each cell to be kept secret, having a large value in general, is accompanied by another cell with a very small value on the row as well as on the column.

A simple RAS procedure will not generate the original high and low value, but two values somewhere in between, which is beyond any reality. Therefore, the RAS procedure is done with starting values for the empty cells (except for the real zeros and insignificant values) derived from another source.

These starting values represent the original distribution of high and low values – even the extreme values. Then, the RAS procedure is generating realistic estimates for the empty cells.

This procedure is started for 2002 and labor (full time equivalent), since there is a very good source that is giving starting values for the empty cells (LISA). LISA is also giving data on labor (working persons) and assumed to give a realistic starting values. Then, 2002 was estimated properly.

Accordingly, the values for 2002 were used as the starting values for the preceding year and the following years. The only thing that requires a RAS procedure, is the economic growth for the empty cells (a little increase or decrease). That is just a fraction of the total missing value.

The last step was to aggregate the 37 sectors from the REJ into the 14 sectors for RAEM.

### V.2.3 Transport data for the Netherlands

The RAEM model uses both passenger and freight transport data. For each pair of NUTS3-regions the number of trips, the monetary and time costs are needed as an input.

The passenger data originates the OVG (Onderzoek VerplaatsingsGedrag). The base year for the passenger data is currently still 2002. However, in a later stage an update will be made to 2006. Many people were asked to register their trip details like origin zone, destination, trip purpose, mode, departure time and arrival time. For RAEM, this data has been aggregated to origin/destination

(OD)-matrices on the level of NUTS3 regions. For each OD-pair and trip purpose the number of trips, the average travel time (hours) and the average distance (km) have been computed. Based on these outcomes the average travel costs for each OD-pair were calculated by multiplying the average travel time with a value of time and by multiplying the average distance with average costs per kilometer. The following trip purposes are used: 1) Commuting, 2) Business, 3) Services/personal care, 4) Shopping, 5) Education, 6) 6 Visits, 7) Other social recreation, 8) Round trips/hiking and 9) Other.

The freight data originates from the national bureau of statistics of the Netherlands (CBS). This data was edited and used in the freight model SMILE. From this model the total number of tonnes transported between each pair of NUT3-regions in 2002 was obtained as well as the average costs per ton. The 2002 data is used as base year data for RAEM.

Finally, since RAEM 3 is a recursive dynamic model, time series data (data for 10 different years) will be used to refine the calibration of the model. This will be done in a later stage.

### V.2.4 Auxiliary data

For the immigration component of the RAEM model an OD-matrix with the migration flows between the different NUTS3 regions is necessary. This OD matrix for the year 2006 was received from the Statistical Office of the Netherlands.

Recursive dynamic part of the model is based on the forecasted national GDP growth provided by the Central Planning Bureau (CPB) of the Netherlands for the time period 2006-2020. Recursive dynamic part of RAEM 3.0 includes the distribution of the total savings between sectors located in the 40 Dutch regions in the form of sector-specific investments. The allocation rule takes into account the average historical growth rates of investments in those sectors. The annual average historical growth rates of investments are calculated for each of the RAEM sectors in the 40 regions of the model based on the time series for the period 1970-2001.

### V.2.5 Armington elasticities of substitution of international trade

Armington elasticities (Armington, 1969) are based on the differentiation of products with respect to their origin and the imperfect substitution in demand between imports and domestic supply. They are critical parameters which, along with the model structure, data and other parameters, determine the results of policy experiments. In the RAEM model we use the substitution elasticities to model the interaction of the Dutch economy with the rest of the world, through imports and exports.

A good estimation of these parameters can improve the quality of the predictions of the model. Nevertheless, papers treating their estimation are rather scarce. In general equilibrium modeling the Armington elasticities are often calibrated on an initial dataset or are even outward guesses.

Common substitution elasticities used in literature range from around 1 to 2. This is supported by some econometric studies, estimating the elasticities at industry or product level. (Kapuscinski C.& Warr P.G.,1996; Saito M.,2004; Reinert K., Ronald-Holst D. 1992). But the reported estimation can differ largely according to the product or trade regime. Empirical work on trade liberalizations, as well as cross section regressions relating trade patterns to tariff and non-tariff barriers, find Armington elasticities that range from about 6 to 15, similar to the ones needed in applied GE models. (Ruhl K.J, 2003)

It may be of no surprise that econometric estimation of substitution elasticities, in the case of trade liberalization, will be higher due to the impact on the structure of the economy. It also appears that these very high substitution elasticities are commonly found for products specifically oriented at export or import, for example certain agricultural products.

The problem with many of the econometric work into substitution elasticities lies in the complex nature of international trade. First of all, data is often quite limited or incomplete. Second, structural changes in the economy may be influencing our estimation of the price (substitution) effect.

Third, as most of these data are time series and have a non-stationary nature, common econometric techniques such as OLS (ordinary least squares) can lead to a phenomenon called 'spurious regression'. In this case, we get misleading results, as statistical tests predict high correlations between actually completely unrelated parameters due to a common trend.
It seems that only recently this problem has been treated in the literature around the econometric estimation of the Armington elasticities. Before, only simple ordinary least squares was used for the estimation of the elasticities, not taking into account the problem of serially correlated errors.

We will perform a set of econometric tests, both the OLS technique and techniques that were suggested in other papers to avoid these specification errors. We will also show that the estimations for the substitution elasticity can vary a lot according to the methodology.

Based on the Armington assumption on international trade and by using the CES functional form for the representation of substitution possibilities between imports and domestically produced goods,, we can derive the following relation between domestic production and imports.

$$\frac{X_m}{X_d} = A \cdot \left( \frac{P_d}{P_m} \right)^{\sigma} \tag{71}$$

(Where $X_m$ and $X_d$ denote import volumes and domestic production and $P_m$ and $P_d$ denote import prices and domestic prices respectively, $\sigma$ represents the Armington elasticity.) This formula is not convenient for an econometric test, but an easy log transformation of the variables makes it possible to have a very convenient formula, that can be tested by linear regression.

$$\log\left( \frac{X_m}{X_d} \right) = a + \sigma \log\left( \frac{P_d}{P_m} \right) + \varepsilon \tag{72}$$

($\varepsilon$ denotes an error term, assumed to be normally distributed with mean 0).

This formula represents the common OLS technique to estimate these parameters, check that this specification completely ignores the time series character of the variables.
A different, but related way to estimate the parameters, is by adapting this formula by adding a first lag of the dependent variable (Kapuscinski C.& Warr P.G.,1996). This is called the partial adjustment model.

$$\log\left( \frac{X_m}{X_d} \right)_t = a + \sigma \log\left( \frac{P_d}{P_m} \right)_t + \eta \log\left( \frac{X_m}{X_d} \right)_{t-1} + \varepsilon_t \tag{73}$$

(the t and t-1 denotes the time and lag on these variables, $\varepsilon_t$ is a 'white noise' error term)

This looks like only a small change on the original model, but the difference is much broader than it seems. Note that in this case the time series character of the variables enters the equation, making it possible that variables are correlated across time. Although the partial adjustment model is an improvement on the static equation, the formulation of the estimable model in levels of variables is likely to lead to statistical problems such as high intercorrelations between regressors (Kapuscinski C.& Warr P.G.,1996).

Probably a better way to improve the static OLS equation is by using the Engle and Granger (1987) class of *error correction models (ECM)*. These models specifically use the long term character of time series and put forward a very different approach. (Kapuscinski C.& Warr P.G.,1996 ; Katherine L. Gibson, 2003)

$$\Delta\log\left(\frac{X_m}{X_d}\right)_t = a + \sigma\Delta\log\left(\frac{P_d}{P_m}\right)_t + \eta\log\left(\frac{X_m}{X_d}\right)_{t-1} + \chi\log\left(\frac{P_d}{P_m}\right)_{t-1} + \varepsilon_t \qquad (74)$$

These classes of models use the first differences on the variables (as it is suggested by econometric theory that this can often transform non-stationary into stationary variables) and add both a lag on the independent and dependent variables as a means to 'correct' the model. We note that this is the unrestricted form of the ECM model, the restricted form would treat the difference between the lag on dependent and independent variable as 1 variable.

To provide the RAEM model with estimates of the Armington elasticities of substitution we have performed an econometric analysis. Our econometric analysis was based on the data on international trade, domestic production and prices from Central Bureau of Statistics (CBS) Netherlands for the period 1980-2002. All the data from CBS was provided in the form of the indices with the year 1996 used as the base year.

To have sufficient data to perform an econometric test, we both need volume indices as price indices. Data on exports was much better specified by products, differencing into a variety of goods. (Total exports, minerals, manufacturing, food, textiles, petroleum, chemicals, metals, others).

Data on imports was limited to general classes. (Total imports, minerals and intermediate goods, consumption goods, investment goods, general goods). We tried to overcome this limitation and relate import goods more to the export good classes by using data on domestic production of a variety of goods and relate this to the price and quantity indices of import goods.

| Minerals | Consumption | Investment | General |
|---|---|---|---|
| Wood | Food | Machines | Public |
| Petroleum | Textile | Electronic | |
| Chemicals | Clothes | Machine products | |
| | Paper | | |

**Table 1:** Relations domestic production to general classes of goods

We do not expect especially high substitution elasticities in the case of the Netherlands, as we will look into aggregated sectors and do not expect big structural changes in the Dutch economy. Values for the elasticity for substitution commonly found are then around 1 or even lower. (K.L Gibson, 2003; Kapuscinski C.& Warr P.G.,1996).

To test the quality of our data, as well as the sensitivity of our data to a different specification of the model, we try the OLS, partial adjustment (PAM) and ECM techniques. In a way similar to Kapuscinski C.& Warr P.G.,1996.
We provide a complete overview of the results of these 3 types of models (cfr. down). The estimates for the elasticities of substitution are in each case statistically significant, but the estimate in the OLS case is very different from the PAM and ECM case. We can also note that the P value for the OLS estimation technique is 0, which is unrealistically low, considering the data we use. This is a clear indication that this kind of model setup suffers from spurious regression. If we include the time-series specification of our data, the estimate for the substitution elasticity drops from 1.8869 to

around 0.40. Interestingly, the estimate for the substitution elasticity is very similar in both ECM and PAM specifications, while the effect of the lag on the volumes is completely different.
We conclude from this test that the ECM specification will probably be most appropriate as a means to estimate the Armington elasticities, comparable with the papers that did a similar specification test. (Kapuscinski C.& Warr P.G.,1996; K.L Gibson, 2003). Still, we will compare the results of the OLS test with the results of the ECM test later in this document.

| **OLS Estimation** | | | | | | |
|---|---|---|---|---|---|---|
| | **Estimates** | **Std. Err.** | **t-value** | **P>\|t\|** | **Confidence Interval (95%)** | |
| logPrice \| | **1.886926** | 0.22274 | 8.47 | 0 | 1.423695 | 2.350156 |
| Constant | 0.019355 | 0.0169 | 1.15 | 0.265 | -0.0158 | 0.054508 |
| **PAM Estimation** | | | | | | |
| | **Estimates** | **Std. Err.** | **t-value** | **P>\|t\|** | **Confidence Interval (95%)** | |
| D(logPrice) | **0.3950189** | 0.0923 | 6.89 | 0.001 | 0.2034575 | 0.5865804 |
| L(logVolumes) | *0.8666736* | 0.0432 | 8.84 | 0 | 0.7768989 | 0.9564482 |
| Constant | 0.0122258 | 0.003 | 9.68 | 0 | 0.0058035 | 0.0186481 |
| **ECM Estimation** | | | | | | |
| | **Estimates** | **Std. Err.** | **t-value** | **P>\|t\|** | **Confidence Interval (95%)** | |
| D(logPrice) | **0.405542** | 0.152872 | 2.65 | 0.016 | 0.084369 | 0.726715 |
| L(logVolumes) | -0.13559 | 0.054587 | -2.48 | 0.023 | -0.25027 | -0.02091 |
| L(logPrices) | 0.352722 | 0.116166 | 3.04 | 0.007 | 0.108667 | 0.596777 |
| Constant | 0.018216 | 0.004389 | 4.15 | 0.001 | 0.008994 | 0.027437 |

**Table 2:** Overview of the results from different estimation procedures, D() is indicating a first difference on the variable, L() a first lag on the variable.

The previous test suggests that the ECM set-up is the most useful, but still we wanted to compare our estimates for our full time series, both import products and export products. We use both the OLS formulation and the ECM formulation to produce estimates for the substitution elasticities. We will only report standard errors, t-values, P values and confidence intervals for the elasticity estimates and not for the other coefficients in the models.

| **Import elasticities (ECM)** | | | | | | |
|---|---|---|---|---|---|---|
| **Product** | **Coefficient** | **Std. Err.** | **t-value** | **P>\|t\|** | **Confidence Interval (95%)** | |
| Total import | 0.405542 | 0.152872 | 2.65 | 0.016 | 0.084369 | 0.726715 |
| Food | 0.215133 | 0.453924 | 0.47 | 0.641 | -0.73853 | 1.168791 |
| Textile | 0.843252 | 0.33695 | 2.5 | 0.022 | 0.135347 | 1.551158 |
| Clothes | 0.905163 | 0.417062 | 2.17 | 0.044 | 0.028949 | 1.781376 |
| Wood | 0.037187 | 0.140146 | 0.27 | 0.794 | -0.25725 | 0.331623 |
| Machines | 0.640483 | 0.30561 | 2.1 | 0.051 | -0.00158 | 1.282545 |
| Electronic | -0.10123 | 0.172022 | -0.59 | 0.564 | -0.46264 | 0.260172 |
| Public | -0.01062 | 0.297737 | -0.04 | 0.972 | -0.63614 | 0.614903 |
| Petroleum | 0.610491 | 0.370204 | 1.65 | 0.116 | -0.16728 | 1.388261 |
| Machine products | 0.354354 | 0.091029 | 3.89 | 0.001 | 0.163109 | 0.545598 |
| Paper | 0.08475 | 0.159985 | 0.53 | 0.603 | -0.25137 | 0.420867 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Chemicals | 0.652659 | 0.245212 | 2.66 | 0.016 | 0.137488 | 1.16783 |
| **Import elasticities (OLS)** | | | | | | |
| **Product** | **Coefficient** | **Std. Err.** | **tvalue** | **P>[t]** | **Confidence Interval (95%)** | |
| Total import | 1,886926 | 0,222748 | 8,47 | 0 | 1,423695 | 2,350156 |
| Food | 3,551704 | 0,772321 | 4,6 | 0 | 1,945574 | 5,157834 |
| Textile | 4,716176 | 0,382957 | 12,32 | 0 | 3,919773 | 5,51258 |
| Clothes | 4,776274 | 0,414393 | 11,53 | 0 | 3,914497 | 5,63805 |
| Wood | 1,116526 | 0,158473 | 7,05 | 0 | 0,786965 | 1,446088 |
| Machines | 4,097577 | 0,808955 | 5,07 | 0 | 2,415262 | 5,779891 |
| Electronic | 0,344144 | 0,091079 | 3,78 | 0,001 | 0,154735 | 0,533553 |
| Public | 0,958114 | 0,191177 | 5,01 | 0 | 0,56054 | 1,355688 |
| Petroleum | 1,591763 | 0,214161 | 7,43 | 0 | 1,146391 | 2,037135 |
| Machine products | 0,817648 | 0,105918 | 7,72 | 0 | 0,597379 | 1,037916 |
| Paper | 0,726198 | 0,150638 | 4,82 | 0 | 0,412928 | 1,039467 |
| Chemicals | 0,749443 | 0,167354 | 4,48 | 0 | 0,401411 | 1,097476 |

**Table 3:** Estimates of import substitution elasticities using both ECM and OLS approach

This table gives a complete overview of our estimates using both the ECM and OLS approach. We see that in many cases the estimated coefficient in the ECM case is a lot lower than in the OLS case. Estimates from products like wood, paper, electronics, … have 'lost' their statistical significance. Pointing again at the possibility of spurious regression. Again, the P values we get from the ECM model are much more in line with what can be normally suspected.

| **Export elasticities (ECM)** | | | | | | |
|---|---|---|---|---|---|---|
| **Product** | **Coef.** | **Std. Err.** | **t** | **P>\|t\|** | **Confidence Interval (95%)** | |
| Total Export | 0.205441 | 0.093368 | 2.2 | 0.041 | 0.009281 | 0.4016 |
| Minerals | -0.05007 | 0.05628 | -0.89 | 0.385 | -0.16831 | 0.06817 |
| Manufacturing | 0.137618 | 0.124198 | 1.11 | 0.282 | -0.12331 | 0.398549 |
| Food | 0.294143 | 0.352276 | 0.83 | 0.415 | -0.44596 | 1.034248 |
| Textiles | 0.561134 | 0.408297 | 1.37 | 0.186 | -0.29667 | 1.418934 |
| Petroleum | -0.48649 | 0.321816 | -1.51 | 0.148 | -1.1626 | 0.189619 |
| Chemicals | 0.903965 | 0.282996 | 3.19 | 0.005 | 0.309413 | 1.498517 |
| Metal products | 0.476998 | 0.22816 | 2.09 | 0.051 | -0.00235 | 0.956344 |
| Other | 0.646737 | 0.212562 | 3.04 | 0.007 | 0.200162 | 1.093313 |
| **Export elasticities (OLS)** | | | | | | |
| **Product** | **Coeff** | **StdError** | **t value** | **P** | **Confidence Interval (95%)** | |
| Total Export | 2,07991 | 0 | 8,76 | 0 | 1,586384 | 3 |
| Minerals | **0,005402** | 0,076651 | 0,07 | 0,944 | -0,154 | 0,164808 |
| Manufacturing | 2,290354 | 0,245379 | 9,33 | 0 | 1,780062 | 2,800646 |
| Food | 3,002685 | 0,907177 | 3,31 | 0,003 | 1,116107 | 4,889263 |
| Textiles | 4,401273 | 0,544287 | 8,09 | 0 | 3,269365 | 5,53318 |

| | | | | | |
|---|---|---|---|---|---|
| Petroleum | **0,148847** | 0,149481 | 1 | 0,331 | -0,16202 | 0,459709 |
| Chemicals | 1,963368 | 0,1566 | 12,54 | 0 | 1,637701 | 2,289036 |
| Metal products | 4,88067 | 0,550705 | 8,86 | 0 | 3,735417 | 6,025923 |
| Other | 2,41488 | 0,203938 | 11,84 | 0 | 1,990768 | 2,838992 |

**Table 4:** Table 4 Estimates of export substitution elasticities using both ECM and OLS approach

We can draw similar conclusions for the estimates of the export elasticities. The export elasticities estimated with the ECM approach are even lower than the import substitution elasticities.

Relating the substitution elasticities we estimated tot the RAEM sectors was another though job, as RAEM contains a lot of service sectors. In the end we still needed to generalize a lot of our parameters, but at least we can relate this to our previous econometric tests.
We connected the substitution elasticities per product to a related RAEM sector. When it was unclear which parameter to take, we related the sector to the substitution parameter for total imports or exports. We choose to take a higher substitution elasticity for the sector 10 and 11, as is suggested that these sector have an above average substitution elasticity. (Kapuscinski C.& Warr P.G.,1996; K.L Gibson, 2003; Saito M.,2004)

| Sector code | Sectors | Relation | Import | Export |
|---|---|---|---|---|
| secr01 | Agriculture | Total | 0.405542 | -0.20544 |
| secr02 | Mining and quarrying | Petroleum | 0.610491 | -0.20544 |
| secr03 | Manufacturing | Textiles | 0.843252 | -0.56113 |
| secr04 | Electricity, gas and water supply | Total | 0.405542 | -0.20544 |
| secr05 | Construction | Metalprod | 0.354354 | -0.477 |
| secr06 | Trade and repair consumer services | Total | 0.405542 | -0.20544 |
| secr07 | Hotels, restaurants and café | Total | 0.405542 | -0.20544 |
| secr08 | Transport | Total | 0.405542 | -0.20544 |
| secr09 | Storage and communication | Total | 0.405542 | -0.20544 |
| secr10 | Financial services | Other | 0.646737 | -0.64674 |
| secr11 | Business services, renting, real estate | Other | 0.646737 | -0.64674 |
| secr12 | Public administration (includes defense and collective social security) | Total | 0.405542 | -0.20544 |
| secr13 | Education | Total | 0.405542 | -0.20544 |
| secr14 | Health and social work | Total | 0.405542 | -0.20544 |
| secr15 | Culture, sports, leisure | Total | 0.405542 | -0.20544 |

**Table 5:** Relation of the parameters to the RAEM sectors.

### V.2.6 Elasticities of substitution between capital and labour inputs

An important parameter for the RAEM model is the elasticity of substitution between capital and labour. For the RAEM model it is the intention to utilize unique elasticities for each of the 15 sectors in each of the 40 regions. These elasticities of substitution were calculated for the 15 sectors at the national level under the assumption that the same elasticities of substitution can then be applied to all 40 Dutch regions.

The data used in the estimations comes from the EU-KLEMS project (http://www.euklems.net/). In this project a database was constructed with measures of economic growth, productivity, employment creation, capital formation and technological change at the industrial level for all European Union member states from 1970 onwards, until 2004 currently.

Many specifications are possible for the relation between labour and capital. There is a lot of debate on this subject among economists. The specifications most often used are the Cobb-Douglas functions, that assume an elasticity of substitution of 1 between labour and capital, and the CES functions, that assume a constant elasticity of substitution (i.e. independent of any variables, including time).

For the estimations at hand a simple model, based on the standard CES demand function, was chosen, that has low data requirements. The basic functional form is, with the index *i* denoting the different sectors,

$$\log\left(\frac{L_i}{K_i}\right)=\beta_{1,i}+\beta_{2,i}\cdot\log\left(\frac{PK_i}{PL_i}\right) \tag{75}$$

The coefficient $\beta_{2,i}$ is then the elasticity of substitution $\sigma_i$ between labour and capital in the sector *i*. The variables L, K, PK and PL denote respectively the quantity of labour performed, the quantity of capital available, the price of capital and the price of labour.

In practice, these variables are not directly available and have to be approximated by other, available variables. In the EU KLEMS data the best available proxies are (with their notation in the EU KLEMS data) labour services, volume index, (LAB_QI) for L, and capital services, volume index, (CAP_QI) for K. The prices of labour and capital are not directly available, but can be calculated by dividing the labour services (LAB_QI) by the labour compensation (in million euros) (LAB) and by dividing the capital services (CAP_QI) by the capital compensation (in million euros) (CAP). LAB_QI and CAP_QI are volume indices with 1995 as the reference year. That they are not the actual quantities poses no problem. Due to the logarithm, the 1995 base numbers of the indices reduce to constants which are added to the constant term $\beta_{1,i}$ in the equation, which is of no interest here. The equation now reads

$$\log\left(\frac{LAB\_QI_i}{CAP\_QI_i}\right)=\beta_{1,i}+\beta_{2,i}\cdot\log\left(\frac{CAP\_QI_i}{LAB\_QI_i}\cdot\frac{LAB_i}{CAP_i}\right) \tag{76}$$

One important point to consider is that the functional form of the relation between labour and capital is not necessarily the same for each sector. There is no information beforehand about what functional form is correct for a sector. A misspecified model will most of the time lead to biased estimates. Since there is no a priori information about this issue, nor does theory provide helpful insights in this regard, the models for the sectors are developed individually on basis of the data, with the model described above as a starting point.

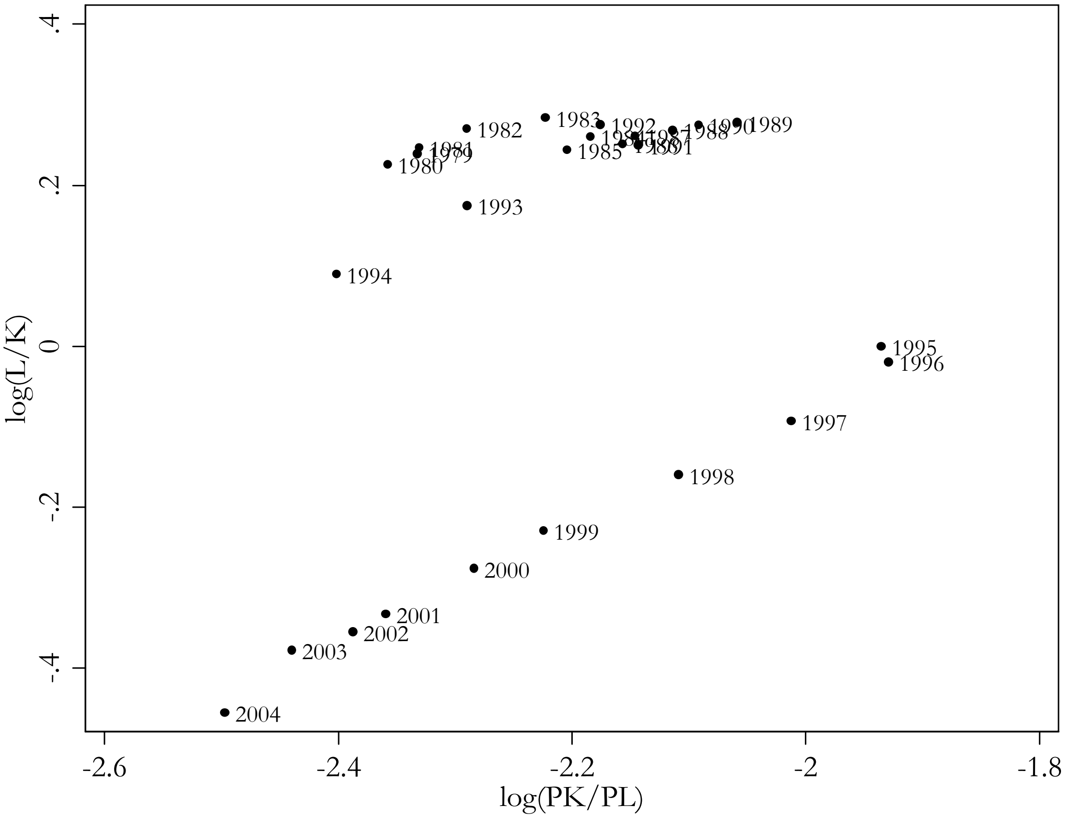

**Figure 3:** Scatter plot of the logarithm of labour on capital vs. the logarithm of the price of capital on the price of labour for the educational sector from the year 1979 to 2004. A shock and possible structural change is clearly visible between the years 1994 and 1995.

After a cursory overview of the data, one can see that in many cases the data does not follow a clear linear relationship (see for example Figure 3). The explanation for this is that some of the individual sectors experienced several (not necessary simultaneous) shocks and structural changes over the years, leading to linear pieces with a different slope connected by transient years. It is for this reason that the data in the estimation was restricted to from 1989 onwards. Due to this restriction, the actual number of data points used in the estimations becomes smaller. Even with this restriction, many of the sectors still contain a shock and maybe a transient. An example is shown in Figure 4.

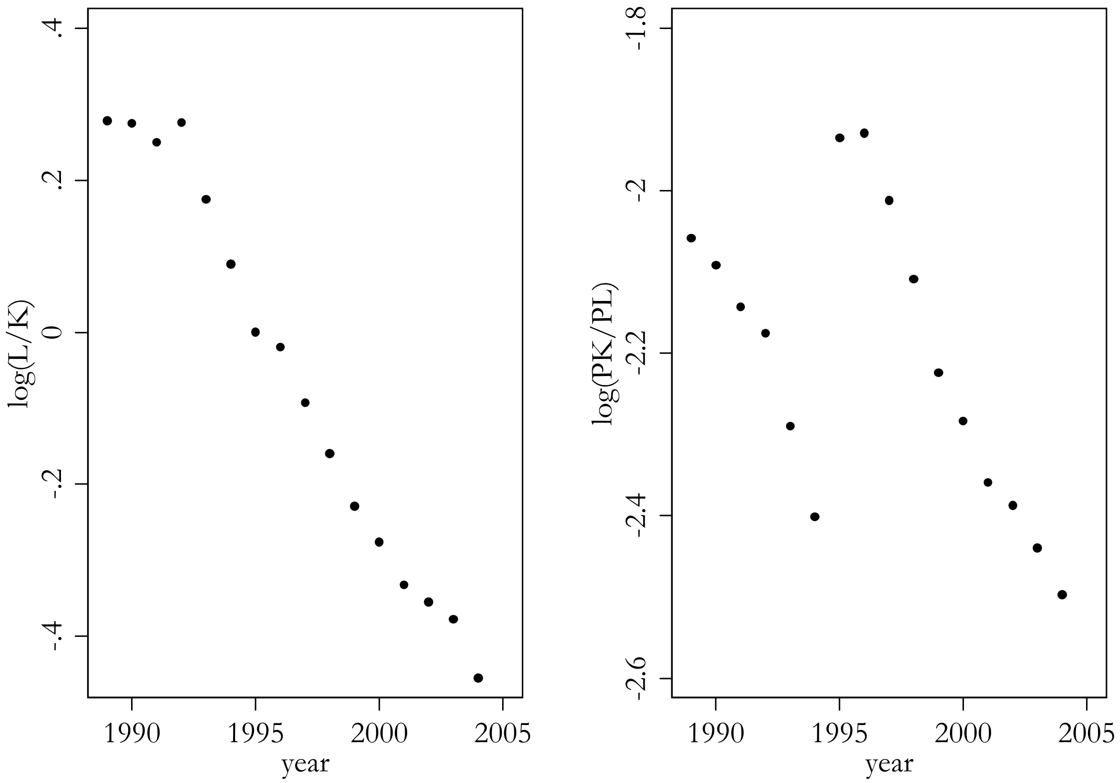

**Figure 4:** The logarithms of labour on capital and the price of capital on the price of labour vs. time for the educational sector. Both have a decreasing trend, but the logarithm of the prices has a discontinuity in the year 1995. Without appropriate modeling, this will give problems in the estimation.

If a sector did experience a shock, this can be remedied in the model by adding a dummy variable for the year that the shock occurred. If the shock also caused a structural change, and hence a change in the relation between labour and capital, an interaction term between the dummy and the logarithm of the prices can capture this change in the elasticity. The true elasticity in this period is then the sum of the coefficients of said logarithm and the interaction. With this interaction the model is of the form, with $I_{year}$ denoting the dummy for the year that the shock occurred

$$\log\left(\frac{L_i}{K_i}\right)=\beta_{1,i}+\beta_{2,i}\cdot\log\left(\frac{PK_i}{PL_i}\right)+\beta_{3,i}\cdot I_{year}+\beta_{4,i}\cdot I_{year}\cdot\log\left(\frac{PK_i}{PL_i}\right) \tag{77}$$

In order to estimate this equation time series analysis is used. This choice of methodology is justified by the fact that the data used for estimation is a time series. Ignoring this aspect might lead to biased estimates and wrong confidence intervals. The time series model used is the ARMA(p,q) model, which assumes an error structure with p lags in the autocorrelation and q lags in the moving average part. Unfortunately, no clear guidelines are available as to how many lags to include in either the autocorrelation part of the error structure or either the moving average part of the error structure. Therefore the procedure outlined below was used.

First, an exploratory scatter plot of the data for a given sector is made. Based on this the presence or absence of a shock will be determined and the year in which it occurred. After choosing the functional form in this way, the autocorrelations and partial autocorrelations are studied to determine possible significant lags for the ARMA model. Then a forward selecting methodology is used with these selected lags to build a model that is performs the best in terms of the Bayesian Information Criterion (BIC). This measure is taken because it gives an indication of the quality of the model and not just the fit of the model. This choice is made because the focus here is on inference and the precise determination of the elasticities of substitution and not on prediction. As a starting point a first order auto-regressive, first order moving average process is chosen. Estimations are performed using the STATA software package.

The estimated elasticities of substitution are reported in Table 6:. In the paragraphs below a detailed discussion is given of the models build for the different sectors and any issues encountered. A discussion of the results is then given.

In the data for the agricultural sector a shock is present in the year 1998. Therefore the dummy and the interaction term were added to the model. During the model building it became clear that an outlier in the year 1993 was skewing the estimates. The fit improved considerably after excluding this outlier from the estimation. A few data points had, for unknown reasons, negative values for the capital variable; these were also excluded from the estimation. The best model in terms of BIC is an ARMA(1, 1 2 3) model.

For the mining and quarrying sector the basic model as outlined above proved to be the best in terms of BIC. There were no other issues of note.

In the data for the manufacturing sector a shock is present for the year 1995. Therefore the dummy and the interaction term were added to the model. The best model in terms of BIC is an ARMA(1 2 3 6, 1) model.

In the data for the electricity, gas and water supply sector a shock is present for the year 2000. The addition of an interaction term turned out to be unnecessary and only the dummy was added to the model. The best model in terms of BIC is an ARMA(1 2 4 5, 1 2) model.

In the data for the construction sector a shock is present for the year 2000. The addition of an interaction term turned out to be unnecessary and only the dummy was added to the model. The best model in terms of BIC is an ARMA(1 6, 1 2) model.

In the data for the trade and repair consumerservices a shock is present for the year 2000. Therefore the dummy and the interaction term were added to the model. The best model in terms of BIC is an ARMA(1 5, 1) model.

In the data for the hotels, restaurant and café sector a shock is present for the year 2001. Therefore the dummy and the interaction were added to the model. This did not give good results. The model could only be made well-fitting and with significant coefficients by adding the first lag of the log of the quantities to the explanatory variables. This deviation from the methodology outlined above was unexpected and no reason could be found as to why this was necessary. The best model in terms of BIC is an ARMA(1 2, 1) model.

In the data for the transport sector a shock was present for the year 1997. Therefore the dummy and the interaction term were added to the model. The best model in terms of BIC is an ARMA(1 2 3, 1) model.

For the storage and communication sector the model without the dummy or the interaction was used. The best model in terms of BIC is an ARMA(1 4, 2) model.

For the financial sector the model without the dummy or the interaction was used. The best model in terms of BIC is an ARMA(1 2 4, 1) model.

In the data for the business services sector a shock was present for the year 1995. Therefore the dummy and the interaction term were added to the model. The best model in terms of BIC is an ARMA(1 2 3, 1) model.

For the public administration sector the basic model as outlined above proved to be the best in terms of BIC. There were no other issues of note.

In the data for the education sector a shock was present for the year 1995. Therefore the dummy and the interaction term were added to the model. The best model in terms of BIC is an ARMA(2, 1 2 3) model.

For the health and social work sector the model without dummy or interaction was used. The best model in terms of BIC is an ARMA(1 2, 1) model.

For the culture, sports and leisure sector the model without the dummy or interaction was used. The best model in terms of BIC is an ARMA(1 5, 2) model.

| Sector code | Sector | Elasticity | Z | P(Z) | Confidence interval (95%) | |
|---|---|---|---|---|---|---|
| secr01 | Agriculture | 0.195 | 85.61 | 0 | 0.190 | 0.199 |
| secr02 | Mining and quarrying | 0.175 | 2.49 | 0.013 | 0.037 | 0.313 |
| secr03 | Manufacturing | 1.001 | 66.71 | 0 | 0.971 | 1.030 |
| secr04 | Electricity, gas and water supply | 0.260 | 8.13 | 0 | 0.197 | 0.322 |
| secr05 | Construction | 0.287 | 9.98 | 0 | 0.231 | 0.344 |
| secr06 | Trade and repair consumerservices | 0.146 | 2.04 | 0.041 | 0.006 | 0.287 |
| secr07 | Hotels, restaurant and café | 0.138 | 2.32 | 0.021 | 0.021 | 0.254 |
| secr08 | Transport | 0.360 | 50.01 | 0 | 0.345 | 0.374 |
| secr09 | Storage and communication | 0.256 | 5.67 | 0 | 0.167 | 0.344 |
| secr10 | Financial services | 0.650 | 28 | 0 | 0.604 | 0.695 |
| secr11 | Business services, renting, real estate | 0.424 | 25.07 | 0 | 0.390 | 0.457 |
| secr12 | Public administration | 0.822 | 64.8 | 0 | 0.797 | 0.847 |
| secr13 | Education | 0.714 | 667.48 | 0 | 0.712 | 0.716 |
| secr14 | Health and social work | 0.125 | 38.34 | 0 | 0.118 | 0.131 |
| secr15 | Culture, sports and leisure | 0.090 | 4.26 | 0 | 0.049 | 0.132 |

**Table 6:** Table with the estimated elasticities of substitution of the different sectors. Note the high p-values and the fairly wide range of the estimates.

The estimated elasticities of substitution can be found in Table 6. It is a bit remarkable that most of these are very significant at the 95% level. This might be a consequence of the small number of observations on which the estimations were performed, as this violates the assumption of a sufficiently large sample inherent in a Wald test, which is the test used here for the reported p-values. It is interesting to note that the spread of the values is quite wide, though they are all within the expected range of 0 to 1.

If the elasticities are weighted by the total outputs per sector an average elasticity of 0.548 is found. This value is not that far from value of 0.7 for the elasticity of substitution between labour and capital that is reported for the country level in Abrue Pessoa et al. (2005). Other estimates are also reported (going from 0.22 to 1). This is a result of the methodological differences between authors, as already mentioned above.

The validation of the individual estimates for the sectors is more difficult, as substantially less work has been perfumed in this regard. Some papers do report sectoral estimates that differ as much as in the table Table 6. The elasticity of 1 for the manufacturing sector is similar to the result of Douglas (1948), but papers following a methodology different from the one described here often report quite different results for the sectoral elasticities (see Balistreri et al. (2003)).

### V.2.7 Reading the model database

The model database is stored in the following excel files:

- File called SAM_2006.xls contains the national level SAM for the Netherlands constructed for the base year 2006
- File called RAEM_transport2006.xls contains the data on passenger trips and freight transportation flows between the 40 Dutch regions and the associated monetary and time transport costs per each pair of regions. Passenger trips are differentiated between the following trip purposes: commuting trips, education trips, shopping trips and other trips.
- File called RAEM_auxiliary2006.xls contains the data on migration, commuting, wages and unemployment on regional level for the year 2006 as well as the elasticities of substitution and other parameters used in the model

The model database is read from Excel during the execution stage inside the GAMS program.

## *V.3 Description of the nonlinear-programming method for estimation of regional input-output (IO) tables*[4]

Our model builds upon earlier work by Wilson (1970) and Batten (1982) with two important departures. First, it explicitly incorporates interregional trade flow information into both the accounting framework and initial estimates of an Inter-Regional Input-Output (IRIO) account. We find this greatly enhances the accuracy of estimation results. Second, the IRIO account is simplified to a Multi-Regional Input-Output (MRIO) account and estimated first, which substantially reduces the possibility of introducing spurious information in lieu of survey data and also diminishes the "dimension explosion" problem in real world applications.

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[4] Based on Canning and Wang (2005)

### V.3.1 General Assumptions and Mathematical Notations

Consider a national economy consisting of N sectors that are distributed over G geographic regions. The sectors use each other's products as inputs for their own production, which is in turn used up either in further production or by final users. Each region exports some of its products to other regions and some to other nations. They also import products from other regions and nations to meet their intermediate and final demand. Assuming a predetermined location of production that defines the structure of the national economic system of regions, the deliveries of goods and services between regions are determined by imbalances between supply and demand inside the different regions.

In this economy, a comprehensive account of annual product and payment flows within and between regions is summarized by an IRIO table. The notation used to describe the elements of a commodity based IRIO table and its relationship to both a national I-O table and to a MRIO table are as follows (expressed in annual values):

Regional gross output, final demands, value added, and international trade:

$x_i^r$ = Gross output of commodity 'i' in region 'r'

$v_i^r$ = Value added by production of commodity 'i' in region 'r'

$y_i^r$ = Final demand (excluding exports) of commodity 'i' in region 'r'

$e_i^r$ = Exports of commodity 'i' from region 'r' to international market

$m_i^r$ = Imports of commodity 'i' to region 'r' from international market

Interregional and international deliveries:

$z_{ij}^{sr}$ = Deliveries of domestic commodity 'i' produced in region 's' for use by sector 'j' in region 'r'

$y_{ik}^{sr}$ = Deliveries of domestic commodity 'i' produced in region 's' for type 'k' final use in region 'r'

$m_{ij}^r$ = International imports of commodity 'i' for use by sector 'j' in region 'r'

$m_{ik}^r$ = International imports of commodity 'i' for type 'k' final use in region 'r'

National Input-Output table (IO):

$x_i$ = Gross domestic output of commodity 'i'

$y_i$ = Final domestic demand (excluding exports) of commodity 'i'

$e_i$ = International exports of commodity 'i' from domestic origins of movement

$v_i$ = Value added by domestic production of commodity 'i'

$z_{ij}$ = Intermediate demand of commodity 'i' by sector 'j'

$m_i$ = Imports of commodity 'i' from international origins of movement

Aggregation variables for linkage to a MRIO table:

$d_i^{sr}$ = Deliveries of domestic commodity ‘i’ from region ‘s’ to region ‘r’

$z_{ij}^{\bullet r}$ = Intermediate demand of commodity ‘i’ by sector ‘j’ in region ‘r’

## V.5.2 IRIO Account and Estimation Model

Using notations defined above, the following two accounting identities describe the relationship among elements of each row (i,r) and column (j,s) of the IRIO table for a static national system of economic regions:

$$\sum_{s=1}^{g} \sum_{j=1}^{n} z_{ij}^{rs} + \sum_{s=1}^{g} \sum_{k=1}^{h} y_{ik}^{rs} + e_i^r = x_i^r \tag{78}$$

$$\sum_{r=1}^{g}\sum_{i=1}^{n} z_{ij}^{rs} + \sum_{i=1}^{n} m_{ij}^{s} + v_j^s = x_j^s \tag{79}$$

At each given year equations (78) and (79) must hold for all i,j ∈ N, k ∈ H and s,r ∈ G, where H is the set of the economic agents. In addition, this IRIO account has to be consistent with a national IO account and related regional economic statistics, which requires the following accounting identities also to be satisfied each year:

$$\sum_{h=1}^{k}\sum_{s=1}^{g} y_{ih}^{sr} + \sum_{k=1}^{h} m_{ih}^{r} = y_i^r \tag{80}$$

$$\sum_{j=1}^{n} m_{ij}^{r} + \sum_{h=1}^{h} m_{ih}^{r} = m_i^r \tag{81}$$

$$\sum_{r=1}^{g} \left( \sum_{s=1}^{g} z_{ij}^{sr} + m_{ij}^{r} \right) = z_{ij} \tag{82}$$

$$\sum_{r=1}^{g} x_i^r = x_i \tag{83}$$

$$\sum_{r=1}^{g} v_i^r = v_i \tag{84}$$

$$\sum_{r=1}^{g} y_i^r = y_i \tag{85}$$

$$\sum_{r=1}^{g} e_i^r = e_i \tag{86}$$

$$\sum_{r=1}^{g} m_i^r = m_i \tag{87}$$

Collectively, equations (78) to (87) define a commodity based IRIO account within a national system of regions. The economic meanings for each of the 10 equations are straightforward. Equation (78) shows that total gross output of commodity ‘i’ in region ‘r’ is delivered to domestic intermediate and final users in all regions (including itself) within the nation and what is not delivered to domestic users is exported to the international market. Equation (79) defines the value of gross output for commodity ‘j’ in region ‘s’ as the sum of the values from all of its intermediate (domestic plus imported) and primary factor inputs. Equation (80) indicates that each region’s total final demand for commodity ‘i’ must be met by final goods and services delivered from all regions within the nation plus imports from other nations, while Equation (81) states each region’s foreign imports of intermediate and final goods and services have to equal the region’s total imports from international markets. Equations (82) to (87) state simply that in a national system of regions, sums of all the region’s economic activities must equal the totals from the national account.

Assume a national input-output table always exists. There also exists superior statistical data for each regional sector on gross outputs and associated value added, total final demands, and international exports and imports $(x_i^r, v_i^r, y_i^r, e_i^r, m_i^r)$. Then all variables on the right side of equations (78) to (87) listed above can be treated as parameters. With this information, we seek to estimate an IRIO table containing G×G different intermediate transaction tables ($Z_{rs}$, $r,s \in G$), 2×G different international transaction tables ($MI_r$, $MY_r$, $r \in G$), and G×G different final demand tables ($Y_{rs}$, $r,s \in G$).

To formulate a mathematical programming model to this problem, one can construct either informed (e.g., survey based) or uninformed (e.g., data pooling) initial estimates for each endogenous element of the IRIO table — $\overline{z}_{ij}^{sr}$ , $\overline{y}_{ik}^{sr}$ , $\overline{m}_{ij}^{r}$ , and $\overline{m}_{ik}^{r}$ — along with reliability measures to weight each initial estimate — $wz_{ij}^{sr}$ , $wy_{ik}^{sr}$ , $wm_{ij}^{r}$ , and $wm_{ik}^{r}$ , and specify a cross-entropy (Harrigan & Buchanan, 1984, Golan et al., 1994) or a quadratic objective penalty function subject to equations (78) to (82) as constraints. In this context, “uninformed” initial estimates are derived in the absence of information about variations in row or column structures in the target account. In such cases, one typically adopts proportional allocation methods and assigns weights in these same proportions. Applying “informed” initial estimates requires the development of a maximum concordance among data sources that support initial estimates. In other words, an informed mathematical programming calibration of an IRIO account requires a classification of sectors and regions that allows using the greatest amount of primary information from multiple sources that collectively provide consistent descriptions of all row or column structures in the target account. Ideally, the primary information sources include statistical measures of reliability that can be used to weight these initial estimates.

For example, the quadratic objective penalty function for this mathematical programming model is as follows:

$$\text{Min}\, S = \frac{1}{2}\ \{\sum_{s=1}^{g}\sum_{r=1}^{g}\ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(z_{ij}^{sr}-\overline{z}_{ij}^{sr})^2}{wz_{ij}^{sr}}+\sum_{s=1}^{g}\sum_{r=1}^{g}\ \sum_{i=1}^{n}\ \sum_{k=1}^{h}\ \frac{(y_{ik}^{sr}-\overline{y}_{ik}^{sr})^2}{wy_{ik}^{sr}}$$

$$+\sum_{r=1}^{g}\ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(m_{ij}^{r}-\overline{m}_{ij}^{r})^2}{wm_{ij}^{r}}+\sum_{r=1}^{g}\sum_{i=1}^{n}\sum_{k=1}^{h}\ \frac{(m_{ik}^{r}-\overline{m}_{ik}^{r})^2}{wm_{ik}^{r}}\} \tag{88}$$

A solution to this quadratic programming model provides a complete set of estimates for a full-fledged IRIO table with imports endogenous. This type of model becomes operational and provides better empirical estimation results on interregional shipments only when interregional trade flow information is explicitly incorporated into both the initial estimates and the underlying accounting framework.

In practice, calibration of such an account directly is hampered by two limitations. First, as combinations of sectors and regions increase, the dimension of this model becomes very large even for a moderate account size. One quickly encounters the problem know as dimension explosion. Related to this, the data requirements of an IRIO account are daunting. The account requires not only knowing the origin and destination of all product flows, but also every intermediate and/or final use must be specified for all such flows. Few national statistical systems can provide such detailed statistics to support the development of informed initial estimates. Therefore, it is not surprising that uninformed initial estimates were used in Battan's approach.

### V.5.3 MRIO Account and Estimation Model

The IRIO account described in the previous section can be easily reduced to a MRIO account by forming aggregations of $z_{ij}^{sr}$ , $y_{ik}^{sr}$ and $m_{ij}^{r}$ as follows:

$$\sum_{j=1}^{n} z_{ij}^{sr} + \sum_{k=1}^{h} y_{ik}^{sr} = d_{i}^{sr} \tag{89}$$

$$\sum_{s=1}^{g} z_{ij}^{sr} + m_{ij}^{r} = z_{ij}^{sr} \tag{90}$$

Inserting Equation (12) into Equation (78) gives us Equation (14):

$$\sum_{s=1}^{g} d_{i}^{rs} + e_{i}^{r} = x_{i}^{r} \tag{91}$$

Inserting Equation (13) into Equation (79) results in Equation (15):

$$\sum_{j=1}^{n} z_{ji}^{\bullet r} + v_{i}^{r} = x_{i}^{r} \tag{92}$$

It is easy to show that the sum of equation (90) by j over N plus equation (80) equals the sum of equation (89) by s over G plus equation (81). This linear combination of equations (80), (81), (12) and (90) gives equation (93)

$$\sum_{j=1}^{n} z_{ij}^{\bullet r} + y_{i}^{r} = \sum_{s=1}^{g} d_{i}^{sr} + m_{i}^{r} \tag{93}$$

Finally, inserting equation (90) into equation (82) results in equation (94):

$$\sum_{r=1}^{g} z_{ij}^{\bullet r} = z_{ij} \tag{94}$$

Equation (91) indicates that the total gross output of commodity 'i' in region 'r' is delivered to the domestic regions (including its own) and what is left over is exported to other nations. No indication about the type of use is given. Equation (92) indicates that the value of the gross output of commodity 'i' in region 'r' is attributed to the value of all sector 'i' intermediate purchases (regardless of origin) and to the value of services from sector 'i' primary factor inputs. Equation (93) indicates total intermediate and final requirements for commodity 'i' in region 'r' must be met by deliveries from all regions (including from its own) within the nation plus imports from other nations. Thus, equations (91) – (94) plus equations (83) – (87) together also consistently defines an accounting framework for the national system of economic regions, conventionally called a MRIO table in the literature (Miller and Blair, 1985, Isard, et al. 1998). Such an account stops short of assigning specific intermediate or final uses for inter/intra regional product flows, but guarantees that these flows exactly meet all regional demands. Further, because this alternative formulation (Equations (91) to (94)) is mathematically equivalent to equations (78) to (82), a solution to the MRIO account will also be consistent with the IRIO account, so that can be seen as an important intermediate step towards estimating a full-fledged IRIO account. Needless to say, the MRIO account has a much smaller dimension thus significantly reduces the data required and computational difficulties to empirically estimate interregional trade flows and inter-industrial transactions. The smaller information requirements make it more plausible to develop an objective function with informed initial estimates and reliability weights. The use of informed initial estimates is another major motivation underlying this alternative formulation.

## *V.4 Model calibration*[5]

The design of a CGE model requires several steps. First, the structure of the general model is determined. Then, a particular functional form has to be chosen for the production and demand functions. Usually Cobb-Douglas, Linear Expenditure System (LES) or Constant Elasticity of Substitution (CES) specifications are selected for this purpose. Finally, the parameter values for the functional forms must be derived. Ideally, all the parameters in the CGE model may be econometrically estimated, using simultaneous equation estimation methods that take into account the overall model structure. However, given the size of CGE models, the required sophistication of techniques, the identification problems and the lack of data, this procedure is considered infeasible (Gunning and Keyzer, 1995). Therefore, the most commonly used procedure to determine the parameter values is calibration (Mansur and Whalley, 1984). The calibration procedure implies that the parameters of the model are identified on the basis of a single observation of the economy. The economy under consideration is assumed to be in equilibrium, a so-called reference equilibrium or benchmark equilibrium. In practice, the benchmark equilibrium or benchmark data set is a Social Accounting Matrix, constructed from national accounts or other governmental data sources. The calibration procedure ensures that the parameters of the model are specified in such a way that the model will reproduce the initial data set as an equilibrium solution.

Once the parameters are calibrated, the model is complete and different policy changes can be simulated. The parameter values are crucial in determining the results of the policy simulations. A schematic presentation, outlining the calibration procedure and the CGE model use is given in Figure 5 (Shoven and Whalley,1992).

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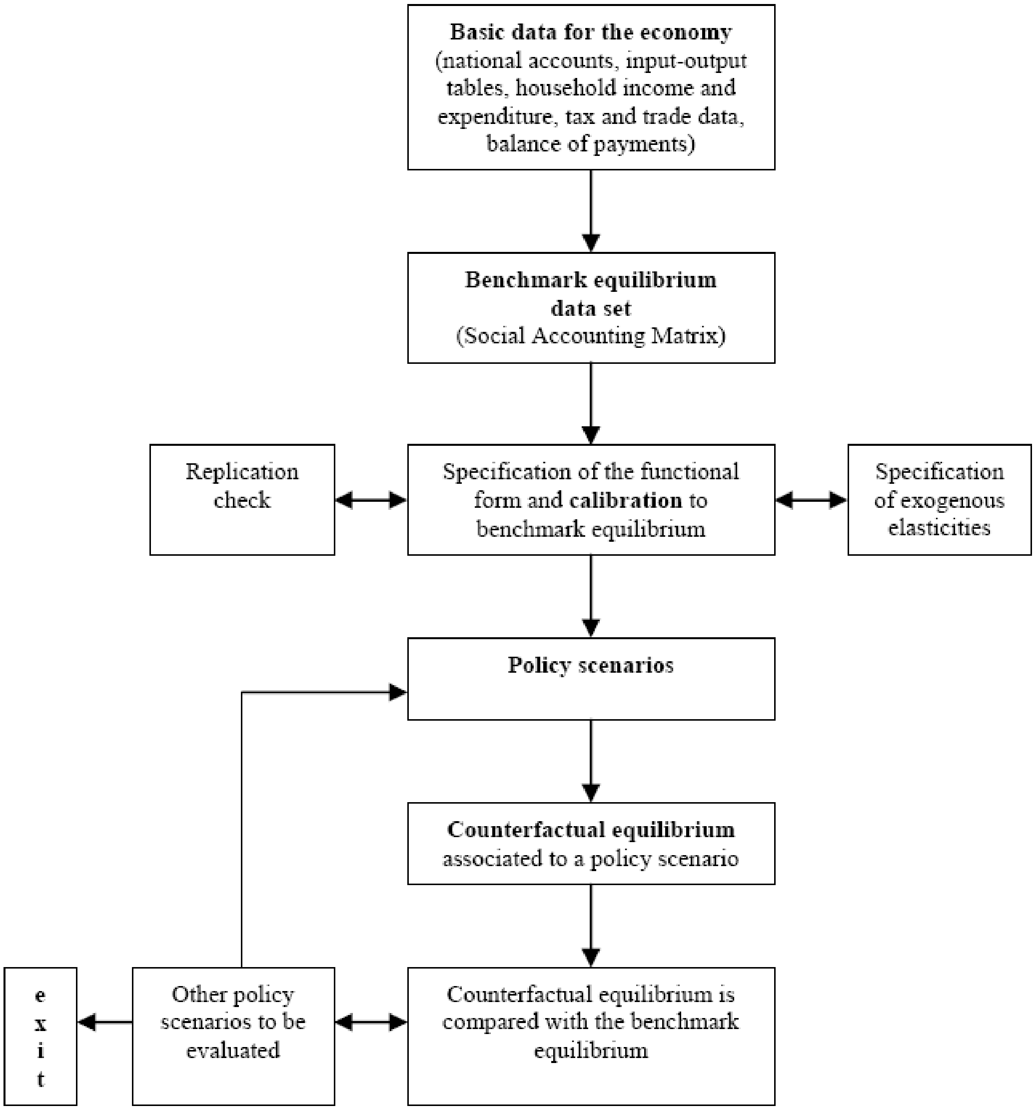
[5] general description of the calibration is based on Mohora (2006)



*Source: Shoven and Whalley, 1992.*

**Figure 5:** A schematic presentation outlining the calibration procedure and the CGE model use

According to figure 6.1, the basic data for the economy are first used to build a consistent framework, the Social Accounting Matrix, which represents the benchmark equilibrium data set for the CGE model. Then, after specifying the functional forms, the model is calibrated to the benchmark equilibrium. For the Cobb-Douglas functions the parameters values are uniquely determined through the calibration procedure, from the Social Accounting Matrix. Instead, for the LES and CES specifications, exogenous elasticities values are also required. The values of these elasticities are usually from the literature. Further, after the calibration of the model, the replication check of the benchmark equilibrium serves as an accuracy test of the computer code of the model. Finally, different policy changes are simulated with the model and the counterfactual equilibrium associated to each of them is compared to the benchmark equilibrium.

### V.4.1 Calibration technique

A CGE model can be written as a system of equations containing a set of unknown parameters $\boldsymbol{\theta}$ , such that a vector of exogenous variables, **X**, produces a vector of endogenous variables **Y** (Adams and Higgs, 1990):

$\mathbf{G(Y,X,\theta,\varepsilon) = 0}$ where $\varepsilon$ is a vector of stochastic disturbances of either known, partially known or unknown distribution. The calibration approach implies that: $\varepsilon = 0$

Therefore, the resulting system of equations can be solved for the vector of parameters $\theta$, using only one observation for the base year. The base year data represent the benchmark equilibrium data set or, more specifically, the Social Accounting Matrix. Parameters whose values cannot be inferred from the benchmark equilibrium data set (such as elasticities of substitution in most cases) are obtained from a search of the literature, or are set arbitrarily.

The restriction $\boldsymbol{\varepsilon = 0}$ indicates that the calibration is interpreted as a non-stochastic approach, as opposed to the stochastic approach of econometrics. This does not mean that economic reality is seen as deterministic when calibrating the CGE models. However, the CGE modeling analyzes the systematic, not the random, responses of economic variables to exogenous stimuli. Thus, ideally, the set of parameters $\theta$ of the CGE model produces the systematic part of the total response in **Y**, with a given **X**.

An important feature of calibration is that the model specification is not statistically tested due to the deterministic procedure of deriving the parameter values from the benchmark data set. Thus the approach uses the key assumption that the benchmark data represent equilibrium for the economy under investigation. Compared to econometric models, which often simplify the structure of the economy to allow for a substantial statistical specification, the calibrated CGE models allow for a richer economic structure (Shoven and Whalley, 1992). Furthermore, because the benchmark data set is usually presented in value terms, to separate price and quantity observations, certain units should be chosen for goods and factors. The most commonly used convention, introduced by Harberger (1962), is to choose units (efficiency units) for both goods and factors such that they have a price of unity in the benchmark equilibrium.

## V.5 Model implementation in GAMS

### V.5.1 Structure of the model code

The RAEM 3.0 mode is implemented using the General Algebraic Modeling System (GAMS). This software is widely used for general equilibrium modeling and has proved to be able to efficiently handle large scale economic models. More information about this software is available from www.gams.com .

The RAEM code starts with the title of the model and short description of the model structure and main elements:

```
$TITLE: RAEM 3.0 (RAEM-TML)
$eolcom #
$TITLE: Regionalized Version - Open Economy Model with one type of Household
Option LIMROW = 0, LIMCOL = 0;
$ontext

    ==========================
    =        SmallOpenEconomy        =
    =                                =
    =              by                =
    =        Dr. Olga Ivanova        =
```

```
=                                          =
=          Transport & Mobility Leuven     =
=                                          =
=        olga.ivanova@tmleuven.be          =
=             www.tmleuven.be              =
==========================================


Purpose:
  Model calibrates parameters of a national economy with the following
  characteristics:
    - one household with nested-LES utility function
    - 15 commodities, used in production and consumption
    - 17 production factors: capital, labor and commodities
    - 15 firms with Leontief technology in value added and
     intermediate aggregate inputs:
       - CES aggregator function for capital and labor
       - Leontief aggregator function in intermediate inputs
    - capital and labor are mobile among sectors
    - national capital and labor endowments are exogenously fixed
    - endogenous saving and investment
    - government sector is included:
       - Cobb-Douglas utility function
       - collects taxes and pays subsidies
    - "small" open economy with Armington CES functions
      for exports and imports
    - commodities are tradable
    - endogenously determined exchange rate/terms of trade
    - disaggregation to NUTS3 level for the Netherlands (40 regions)
    - monopolistic competition
    - commuting decisions based on CPB model - spatial interaction attraction constraint model
      (http://www.cpb.nl/nl/pub/cpbreeksen/cpbreport/2003_1/s2_3.pdf)
    - time and monetary commuting costs are a part of the EV households welfare
    - freight transport costs differentiated by OD pair
    - passenger trips is modeled according to generation-distribution model
    - business trips is modeled according to generation-distribution model
    - passenger trips include shopping, education and other trips
    - migration between the regions based on generation-distribution model described by CPB
     (http://www.ersa.org/ersaconfs/ersa03/cdrom/papers/124.pdf )
    - unemployment is modeled according to the wage curve estimated in
      (http://www.cepii.fr/anglaisgraph/workpap/pdf/2000/wp00-21.pdf )

Short description of the model database:
    - SAM is constructed on the basis of 2006 supply and use tables
    - exports is split between own exports and re-exports using the TNO study of 2001
    - we use national accounts data for 2006
    - number of firms is for 2005
    - Armington elasticities are based on TML's own estimations using time-series analysis
   - Elasticities of substitution between capital and labor are based on TML's own estimations using time-
      series analysis

Notational conventions:
    -scalars, parameters and data are in lower case
    -VARIABLES (and their initial levels) and EQUATION names are in
     CAPITAL letters
    -EQUATION names always begin with EQ
    -initial values of variables and parameters are indicated with Z
     added to their names
```

The main structure of the GAMS code consists of the following elements:

- Definition of the sets
- Importing SAM to GAMS
- Importing regional data to GAMS

- Importing other auxiliary data for GAMS
- Constructing the indicators for the split of national SAM between the 40 Dutch regions
- Importing the data on model parameters estimated using time-series data
- Declaration of scalars and parameters used in the model
- Declaration of parameters, which denote initial values of model variables
- Assigning the initial (base year) values of model variables and checking that all the data is consistent. If this is not the case, the GAMS program is immediately terminated with a corresponding message.
- Estimating interregional trade flows with the nonlinear programming approach (Used under the first run of the program. Resulting data is saved in an Excel file. In further model runs this data is not estimated but read directly from Excel file.)
- Calibration of the model parameters
- Declaration of model variables
- Declaration of model equations
- Formulation of model equations
- Formulation of the RAEM model in non-linear programming and mixed complementarity formats
- Initialization of model variables
- Setting lower bounds for the model variables'
- Exogenously fix the variables, which are initially equal to zero
- Model closure and numeraire
- Declaration of parameters, used for reporting of model results
- Declaration of the time periods set for the recursive dynamic part
- Declaration of investment related parameters
- Loop over the time periods
- Within the time loop: translate total savings into sector-specific investments, initialize model variables, solve the model, calculate the output parameters for each time period
- Write the results of the model into excel file

### V.5.2 Model numeraire

A common assumption for a CGE model, which is also adopted here, is that the economy is initially in equilibrium with the quantities normalized in such a way that the prices are equal to unity. Due to the homogeneity of degree zero in prices, the model can only determine relative prices. A particular price has been selected to provide the numeraire against which all the prices in the model will be measured. In the RAEM model, the GDP deflator is chosen as the numeraire and exogenously fixed in the model.

### V.5.3 Closure of the model and exogenously fixed variables

The formal introduction of the concept of closure rule can be traced back to Sen (1963). Sen (1963) showed that the necessary ex-post equality between savings and investment cannot be fulfilled when all the following conditions are satisfied: the factors are paid at their marginal productivity, household consumption is a function of real income, real investment is fixed, and the factors are fully employed. The equilibrium is achieved only by relaxing one of these constrains. The choice of the constraint to be dropped represents in fact the choice of the closure rule.

In mathematical terms, the model should consist of an equal number of independent equations and endogenous variables. The closure rule reflects the choice of the model builder of which variables are exogenous and which variables are endogenous, so as to achieve ex-post equality.

The following variables are exogenously fixed in the RAEM model and define its closure:

- Sector-specific capital endowments
- Governmental transfers and savings
- Transfers from abroad
- Price of labor in the rest of the world and Dutch labor supplied to the rest of the world
- Time and monetary costs of freight transport
- Time and monetary costs of passenger transport by trip purpose

The closure of the RAEM model is thus defined by a particular set of the exogenously fixed variables. The model closure can be changed by the model user under the simulations given the needs of the particular policy analysis and the assumptions made under the simulation. The user should take care that the number of variables exogenously fixed in the model does not change. This ensures that the number of unknowns in the nonlinear system of equations is equal to the number of equations.

### V.5.4 How to implement simulations with the model

Under the simulations with RAEM 3.0 model one can change any of the exogenously fixed model variables listed in the previous paragraph as well as all model parameters (both estimated and calibrated). Model parameters, which are usually used to set up the simulation with the RAEM model, include consumer preference parameters, producer technology parameters and governmental taxes and subsidies.

## *V.6 Output of the model*

Output of the RAEM 3.0 model is calculated as the percentage changes in the model variables, relative to the baseline values. RAEM 3.0 output includes relative changes in the following variables:

| Name of the excel sheet with model output | Description of the model variables |
|---|---|
| P (%) | Domestic sales prices of commodities |
| PD (%) | Domestic producer unit costs |
| PDDT (%) | Composite prices of domestic commodities |
| PDC (%) | Producer prices of commodities (under monopolistic competition) |
| ER(%) | Exchange rate/ terms of trade |
| INDEX(%) | Consumer price index |
| PI(%) | Price of the composite investment good |

| Name of the excel sheet with model output | Description of the model variables |
|---|---|
| PMEU25(%) | Price of imports from EU25 in local currency |
| PMROW(%) | Price of imports from the rest of the world in local currency |
| PLROW(%) | Price of labour supplied to the rest of the world (exogenous) |
| PL(%) | Domestic region specific price of labour |
| RK(%) | Return to capital |
| RGD(%) | Real interest rate |
| LS(%) | Regional labour endowment (exogenous) |
| LROW(%) | Labour supplied to the rest of the world (exogenous) |
| X(%) | Domestic sales (commodities of both domestic and foreign origin) |
| XD(%) | Gross domestic output per sector |
| XDDE(%) | Domestic production delivered to domestic market per origin-destination pair of regions |
| XDD(%) | Gross domestic output delivered to domestic market per region of destination |
| TMX(%) | Commodity needed for production of nationwide freight transport and trade service |
| EEU25(%) | Exports to EU25 |
| EROW(%) | Exports to the rest of the world |
| MEU25(%) | Imports from EU25 |
| MROW(%) | Imports from the rest of the world |
| ET(%) | Total real exports |
| MT(%) | Total real imports |
| IT(%) | Total investments |
| K(%) | Capital inputs |
| L(%) | Labour inputs |
| BTRIPS(%) | Business trips of particular sector per origin-destination pair of regions |
| BTRIPST(%) | Total business trips of the sector |
| BTSHARE(%) | Share of the trade with the region j in the total interregional trade of the region i |
| BTIME(%) | Time costs of the business trips |
| BMONT(%) | Monetary costs of the business trips |
| C(%) | Demand for the composite consumer goods |
| CBUD(%) | Household’s consumption budget |

| Name of the excel sheet with model output | Description of the model variables |
|---|---|
| Y(%) | Household's income |
| SH(%) | Household's savings |
| SG(%) | Government's savings |
| SEU25(%) | Investments received from EU25 (exogenous) |
| SROW(%) | Investments received from the rest of the world (exogenous) |
| S(%) | Total domestic savings |
| I(%) | Demand for investment goods |
| CG(%) | Demand for commodities of the federal government |
| TAXR(%) | Overall tax revenues of the federal government |
| SUBS(%) | Total subsidies of the federal government |
| TRF(%) | Total transfers from federal government to households (exogenous) |
| TREU25(%) | Total transfers to federal government from EU25 (exogenous) |
| GDP(%) | Real gross domestic product |
| GDPC(%) | Nominal gross domestic product |
| GDPDEF(%) | GDP deflator (exogenous-numeraire) |
| INDEXE (%) | Price index for exports |
| INDEXM (%) | Price index for imports |
| PTM (%) | Composite price of nationwide freight trade and transport service |
| PEV (%) | Price of the unit of the household's utility |
| SII (%) | Budget associated with the household's utility |
| EV | Change in the household's welfare calculated as the percentage of the equivalent variation measure in the total income of the household |
| U (%) | Utility level of the household |
| SV (%) | Changes in stocks |
| NF (%) | Equilibrium number of the monopolistic firms |
| AUXV (%) | Auxiliary variable |
| PROFITS (%) | Profits of the sectors |
| PW (%) | Composite wage of the regional household |
| LMIG (%) | Labour migration per origin-destination pair of regions |
| UNEMP (%) | Regional unemployment level |
| UNRATE (%) | Regional unemployment rate |

| Name of the excel sheet with model output | Description of the model variables |
|---|---|
| LCM (%) | Commuters per origin-destination pair of regions |
| SHOPTRIPS (%) | Shopping trips |
| OTHTRIPS (%) | Other/travel trips |
| EDUTRIPS (%) | Education trips |

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# VII Mathematical formulation of the model

| **RAEM sectors** | |
|---|---|
| 1 | Agriculture |
| 2 | Mining and quarrying |
| 3 | Manufacturing |
| 4 | Electricity, gas and water supply |
| 5 | Construction |
| 6 | Trade and repair consumer services |
| 7 | Hotels, restaurants and café |
| 8 | Transport |
| 9 | Storage and communication |
| 10 | Financial services |
| 11 | Business services, renting, real estate |
| 12 | Public administration (includes defence and collective social security) |
| 13 | Education |
| 14 | Health and social work |
| 15 | Culture, sports, leisure |

| **Regions** | |
|---|---|
| 1 | Oost-Groningen |
| 2 | Delfzijl en omgeving |
| 3 | Overig Groningen |
| 4 | Noord-Friesland |
| 5 | Zuidwest-Friesland |
| 6 | Zuidoost-Friesland |
| 7 | Noord-Drenthe |
| 8 | Zuidoost-Drenthe |
| 9 | Zuidwest-Drenthe |
| 10 | Noord-Overijssel |
| 11 | Zuidwest-Overijssel |
| 12 | Twente |

| | |
|---|---|
| 13 | Veluwe |
| 14 | Achterhoek |
| 15 | Arnhem/Nijmegen |
| 16 | Zuidwest-Gelderland |
| 17 | Utrecht |
| 18 | Kop van Noord-Holland |
| 19 | Alkmaar en omgeving |
| 20 | IJmond |
| 21 | Agglomeratie Haarlem |
| 22 | Zaanstreek |
| 23 | Groot-Amsterdam |
| 24 | Het Gooi en Vechtstreek |
| 25 | Agglomeratie Leiden en Bollenstreek |
| 26 | Agglomeratie 's-Gravenhage |
| 27 | Delft en Westland |
| 28 | Oost-Zuid-Holland |
| 29 | Groot-Rijnmond |
| 30 | Zuidoost-Zuid-Holland |
| 31 | Zeeuwsch-Vlaanderen |
| 32 | Overig Zeeland |
| 33 | West-Noord-Brabant |
| 34 | Midden-Noord-Brabant |
| 35 | Noordoost-Noord-Brabant |
| 36 | Zuidoost-Noord-Brabant |
| 37 | Noord-Limburg |
| 38 | Midden-Limburg |
| 39 | Zuid-Limburg |
| 40 | Flevoland |

In the following formulas the indices *i*, *j*, *k* and *l* denote regions and the indices *Si, Sj*, and *Sk* denote sectors. The total number of the regions equals N = 40. Subscript '0' in the formulas below refers to the initial baseline values of the model variables.

| Model Parameter | Description |
|---|---|
| $\text{frisch}_{i}$ | initial value of Frisch parameter in nested-LES utility function |
| $\text{mps}_{i}$ | marginal propensity to save of households |
| $\text{elasLS}_{i}$ | real wage elasticity of labour supply |
| $\text{elasY}_{Si,i}$ | Initial income elasticities of demand |
| $\text{elasReg}_{Si,i}$ | Demand elasticity between the varieties produced by the firms |
| $\text{fcReg}_{Si,i}$ | Share of fixed costs in total ones |
| $\text{fcK}_{Si,i}$ | Total capital fixed costs |
| $\text{fcL}_{Si,i}$ | Total labour fixed costs |
| phillips | Phillips parameter of the wage curve |
| $\text{aM}_{j,i}$ | Scale parameter of the matching function |
| $\alpha\text{M}_{j,i}$ | Share parameter of the matching function related to vacancies |
| $\beta\text{T}_{j,i}$ | Travel costs related parameter of the matching function |
| $\text{trep}_{i}$ | Replacement rate of unemployed |
| $\text{sp}_{Si}$ | Subsidies rate on production |
| $\text{sc}_{Si}$ | Subsidies rate on products |
| $\text{tc}_{Si}$ | Tax rate on products |
| $\text{vatc}_{Si}$ | VAT tax rate on products |
| $\text{exst}_{Si}$ | Tax rate of excise on products |
| $\text{tk}_{Si}$ | Corporate tax rate |
| $\text{tl}_{Si}$ | Tax rate on labour use employers contribution |
| $\text{tl1}_{Si}$ | Tax rate on labour use employees contribution |
| $\text{txd}_{Si}$ | Tax rate on production |
| ty | Income tax rate |
| $\text{io}_{Si,Sj,i}$ | Technical coefficients intermediate inputs |
| $\beta\text{BT}_{Si,i}$ | Share parameter for the generation of business trips |
| $\alpha\text{BT}_{Si,j,i}$ | Scale parameter for the generation of the business trips |
| $\text{delta}_{Si,i}$ | Depreciation rate |
| $\sigma\text{A}_{Si,i}$ | Armington elasticity of substitution between imports and domestic commodities |
| $\sigma\text{A1}_{Si,i}$ | Elasticity of substitution between commodities from different regions |
| $\gamma\text{A1}_{Si,i}$ | CES share parameter of Armington function for imports from EU25 |

| Model Parameter | Description |
|---|---|
| $\gamma A2_{Si,i}$ | CES share parameter of Armington function for imports from the rest of the world |
| $\gamma A3_{Si,i}$ | CES share parameter of Armington function for domestic goods |
| $\gamma A4_{Si,j,i}$ | CES share parameter for domestic commodities from different regions |
| $aA_{Si,i}$ | Scale parameter of Armington function |
| $aA1_{Si,i}$ | Scale parameter of CES interregional trade function |
| $\alpha H_{Si,i}$ | Power in nested-LES household utility |
| $\mu H_{Si,i}$ | Subsistence household consumption quantity of commodities |
| $\alpha I_{Si,i}$ | Cobb-Douglas power of the investment production function |
| $\alpha G_{Si,i}$ | Cobb-Douglas power of the government utility function |
| $svs_{Si,i}$ | Inventory shares |
| $atm_{Si,i}$ | Share of commodity used for production of nationwide freight transport and trade services |
| $elasE_{Si}$ | Elasticity of domestic exports |
| $\beta SHOP_{j,i}$ | Share for the generation of the shopping trips |
| $\beta EDU_{j,i}$ | Share for the generation of the education trips |
| $\beta OTHER_{j,i}$ | Share for the generation of travel(other) trips |
| $\alpha TRIPS_{i}$ | Power parameter of the gravity function |
| $Bmig_{i}$ | Taste parameter for generation of total migration flow from region |
| $Amig_{i,j}$ | Taste parameter for the distribution of the generated migration |
| ParW1 | Estimated parameter associated with the log(output divided by employment) |
| ParW3 | Estimated parameter associated with the log(employment rate) |
| $ioKL_{Si,i}$ | technical coefficients for capital-labour bundle |
| $\sigma KL_{Si,i}$ | CES elasticity of substitution between capital and labour |
| $\gamma K_{Si,i}$ | CES share parameter for capital |
| $\gamma L_{Si,i}$ | CES share parameter for labour |
| $aKL_{Si,i}$ | Scaling parameter of the CES function |
| $\alpha T$ | Parameter associated with the available labour force |
| $\gamma T$ | Parameter associated with the distance between regions |
| $scalarT_{Si,i}$ | Scaling parameter calibrated on the initial data on commuting |
| $ParW0_{i}$ | Constant parameter (calibrated) |

| Model Variable | Description |
|---|---|
| $P_{Si,i}$ | Domestic sales prices of commodities |
| $PD_{Si,i}$ | Domestic producer unit costs |
| $PDDT_{Si,i}$ | Composite prices of domestic commodities |
| $PDC_{Si,i}$ | Producer prices of commodities (under monopolistic competition) |
| ER | Exchange rate/ terms of trade |
| $INDEX_i$ | Consumer price index |
| PI | Price of the composite investment good |
| $PMEU25_i$ | Price of imports from EU25 in local currency |
| $PMROW_i$ | Price of imports from the rest of the world in local currency |
| PLROW | Price of labour supplied to the rest of the world (exogenous) |
| $PL_i$ | Domestic region specific price of labour |
| $RK_{Si,i}$ | Return to capital |
| RGD | Real interest rate |
| $LS_i$ | Regional labour endowment (exogenous) |
| $LROW_i$ | Labour supplied to the rest of the world (exogenous) |
| $X_{Si,i}$ | Domestic sales (commodities of both domestic and foreign origin) |
| $XD_{Si,i}$ | Gross domestic output per sector |
| $XDDE_{Si,j,i}$ | Domestic production delivered to domestic market per origin-destination pair of regions |
| $XDD_{Si,i}$ | Gross domestic output delivered to domestic market per region of destination |
| $TMX_{Si,i}$ | Commodity needed for production of nationwide freight transport and trade service |
| $EEU25_{Si,i}$ | Exports to EU25 |
| $EROW_{Si,i}$ | Exports to the rest of the world |
| $MEU25_{Si,i}$ | Imports from EU25 |
| $MROW_{Si,i}$ | Imports from the rest of the world |
| ET | Total real exports |
| MT | Total real imports |

| Model Variable | Description |
|---|---|
| IT | Total investments |
| $K_{Si,i}$ | Capital inputs |
| $L_{Si,i}$ | Labour inputs |
| $BTRIPS_{Si,j,i}$ | Business trips of particular sector per origin-destination pair of regions |
| $BTRIPST_{Si,i}$ | Total business trips of the sector |
| $BTSHARE_{j,i}$ | Share of the trade with the region j in the total interregional trade of the region i |
| $BTIME_{j,i}$ | Time costs of the business trips |
| $BMONT_{j,i}$ | Monetary costs of the business trips |
| $C_{Si,i}$ | Demand for the composite consumer goods |
| $CBUD_i$ | Household's consumption budget |
| $Y_i$ | Household's income |
| $SH_i$ | Household's savings |
| SG | Government's savings |
| SEU25 | Investments received from EU25 (exogenous) |
| SROW | Investments received from the rest of the world (exogenous) |
| S | Total domestic savings |
| $I_{Si,i}$ | Demand for investment goods |
| $CG_{Si,i}$ | Demand for commodities of the federal government |
| TAXR | Overall tax revenues of the federal government |
| SUBS | Total subsidies of the federal government |
| $TRF_i$ | Total transfers from federal government to households (exogenous) |
| TREU25 | Total transfers to federal government from EU25 (exogenous) |
| GDP | Real gross domestic product |
| GDPC | Nominal gross domestic product |
| GDPDEF | GDP deflator (exogenous-numeraire) |
| INDEXE | Price index for exports |
| INDEXM | Price index for imports |
| PTM | Composite price of nationwide freight trade and transport service |
| $PEV_i$ | Price of the unit of the household's utility |

| Model Variable | Description |
|---|---|
| $SII_i$ | Budget associated with the household's utility |
| $EV_i$ | Change in the household's welfare calculated as the percentage of the equivalent variation measure in the total income of the household |
| $U_i$ | Utility level of the household |
| $SV_{Si,i}$ | Changes in stocks |
| $NF_{Si,i}$ | Equilibrium number of the monopolistic firms |
| $AUXV_{Si,i}$ | Auxiliary variable |
| $PROFITS_{Si,i}$ | Profits of the sectors |
| $PW_i$ | Composite wage of the regional household |
| $LMIG_{j,i}$ | Labour migration per origin-destination pair of regions |
| $UNEMP_i$ | Regional unemployment level |
| $UNRATE_i$ | Regional unemployment rate |
| $LCM_{j,i}$ | Commuters per origin-destination pair of regions |
| $Ttime_{j,i}$ | Time costs of commuting |
| $Tmoney_{j,i}$ | Monetary costs of commuting |
| $trm_{Si,j,i}$ | Freight transport margins (share of the freight transportation services used per unit of the transported good) |
| $SHOPTRIPS_{j,i}$ | Shopping trips |
| $SHOPTIME_{j,i}$ | Time costs of shopping trips |
| $SHOPMONT_{j,i}$ | Monetary costs of shopping trips |
| $OTHTRIPS_{j,i}$ | Other/travel trips |
| $OTHTIME_{j,i}$ | Time costs of other/travel trips |
| $OTHMONT_{j,i}$ | Monetary costs of other/travel trips |
| $EDUTRIPS_{j,i}$ | Education trips |
| $EDUTIME_{j,i}$ | Time costs of education trips |
| $EDUMONT_{j,i}$ | Monetary costs of education trips |
| $PKL_{Si,i}$ | Price of composite labour-capital bundle |
| $KL_{Si,i}$ | Capital-labour bundle |

## Model Equations

$$\begin{aligned} P_{Si,i} \cdot X_{Si,i} &= \sum\nolimits_j \left( XDDE_{Si,j,i} \cdot \left( PDC_{Si,j} + PTM \cdot trmV_{Si,j,i} \right) \right) \\ &+ PMEU25_{Si} \cdot MEU25_{Si,i} + PMROW_{Si} \cdot MROW_{Si,i} \end{aligned} \tag{1}$$

$$\begin{aligned} & PD_{Si,i} \cdot XD_{Si,i} \cdot \left(1 - txd_{Si} + sp_{Si}\right) = K_{Si,i} \cdot \left( \left(1 + tk_{Si}\right) \cdot RK_{Si,i} + \delta_{Si,i} \cdot PI \right) \\ & + PL_i \cdot L_{Si,i} \cdot \left(1 + tl1_{Si} + \left(1 + tl1_{Si}\right) \cdot tl1_{Si}\right) + \sum\nolimits_{Sj} \left( io_{Sj,Si,i} \cdot XD_{Si,i} \cdot P_{Sj,i} \right) \\ & \sum\nolimits_j \left( BTRIPS_{Si,i,j} \cdot BMONT_{i,j} \right) \cdot \sum\nolimits_{Sj=transpSj} \left( P_{Si,i} \right) \end{aligned} \tag{2}$$

$$PDDT_{Si,i} \cdot XDD_{Si,i} = \sum\nolimits_j \left( XDDE_{Si,j,i} \cdot \left( PDC_{Si,i} + PTM \cdot rtm_{j,i,Si} \right) \right) \tag{3}$$

$$PDC_{Si,i} = PD_{Si,i} \cdot AUXV_{Si,i} \tag{4}$$

$$INDEX_i = \frac{\sum_{Si=CZSii} C^0_{Si,i} \cdot P_{Si,i} \cdot \left(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\right)}{\sum_{Si=CZSii} C^0_{Si,i} \cdot P^0_{Si,i} \cdot \left(1 - sc^0_{Si} + tc^0_{Si} + vatc^0_{Si} + exst^0_{Si}\right)} \tag{5}$$

$$PEV_i = \prod\nolimits_{Si=CZSii} \left( \left( P_{Si,i} \cdot \left(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\right) \right)^{\alpha H_{Si,i}} \right) \tag{6}$$

$$PI = \prod\nolimits_{Si=\alpha ISii} \prod\nolimits_{i=\alpha ISii} \left( \frac{P_{Si,i} \cdot \left(1 - sc_{Si} + tc_{Si} + vatc_{Si} + exst_{Si}\right)}{\alpha I_{Si,i}} \right)^{\alpha I_{Si,i}} \tag{7}$$

$$RGD_i = \frac{\sum_{Si} \left( RK_{Si,i} \cdot K_{Si,i} \right)}{\sum_{Sj} \left( K_{Sj,i} \right)} \tag{8}$$

$$PMEU25_i = PWMEU25^0_i \cdot ER \tag{9}$$

$$PMROW_i = PWMROW^0_i \cdot ER \tag{10}$$

$$X_{Si,i} = C_{Si,i} + CG_{Si,i} + I_{Si,i} + SV_{Si,i} + TMX_{Si,i} + \sum_{Sj} (io_{Si,Sj,i} \cdot XD_{Sj,i}) \text{ for } Si \neq transport \qquad (11)$$

$$\begin{aligned} X_{Si,i} &= C_{Si,i} + CG_{Si,i} + I_{Si,i} + SV_{Si,i} + TMX_{Si,i} + \sum_{Sj} (io_{Si,Sj,i} \cdot XD_{Sj,i}) \\ &+ \sum_{Sj} \sum_{j} (BTRIPS_{Sj,i,j} \cdot BMONT_{i,j}) + \sum_{j} (Tmoney_{i,j} \cdot LCM_{i,j} \\ &+ SHOPTRIPS_{i,j} \cdot SHOPMONT_{i,j} + OTHTRIPS_{i,j} \cdot OTHMONT_{i,j} \\ &+ EDUTRIPS_{i,j} \cdot EDUMONT_{i,j}) \end{aligned} \quad \text{for } Si = transport \qquad (12)$$

$$XD_{Si,i} = \left(\sum_{j} \left(XDDE_{Si,i,j}\right) + EEU25_{Si,i} + EROW_{Si,i}\right) \cdot AUXV_{Si,i} \qquad (13)$$

$$XDDE_{Si,j,i} = XDD_{Si,i} \cdot \left(\frac{\gamma A4_{Si,j,i}}{PDC_{Si,j} + PTM \cdot trmV_{Si,j,i}}\right)^{\sigma A1_{Si,i}} \cdot \left(PDDT_{Si,i}\right)^{\sigma A1_{Si,i}} \cdot \left(aA_{Si,i}\right)^{(\sigma A1_{Si,i}-1)} \qquad (14)$$

$$XDD_{Si,i} = X_{Si,i} \cdot \left(\frac{\gamma A3_{Si,i}}{PDDT_{Si,i}}\right)^{\sigma A_{Si,i}} \cdot \left(P_{Si,i}\right)^{\sigma A_{Si,i}} \cdot \left(aA_{Si,i}\right)^{(\sigma A_{Si,i}-1)} \qquad (15)$$

$$EEU25_{Si,i} = EEU25^{0}_{Si,i} \cdot \left(\frac{INDEX_{i}}{PD_{Si,i}}\right)^{elasE_{Si}} \qquad (16)$$

$$EROW_{Si,i} = EROW^{0}_{Si,i} \cdot \left(\frac{INDEX_{i}}{PD_{Si,i}}\right)^{elasE_{Si}} \qquad (17)$$

$$TMX_{Si,i} = atm_{Si,i} \cdot \sum_{Sj} \sum_{k} \sum_{j} \left(trmV_{Sj,k,j} \cdot XDDE_{Sj,k,j}\right) \qquad (18)$$

$$MEU25_{Si,i} = X_{Si,i} \cdot \left(\frac{\gamma A1_{Si,i}}{PMEU25_{Si}}\right)^{\sigma A_{Si,i}} \cdot \left(P_{Si,i}\right)^{\sigma A_{Si,i}} \cdot \left(aA_{Si,i}\right)^{(\sigma A_{Si,i}-1)} \qquad (19)$$

$$MROW_{Si,i} = X_{Si,i} \cdot \left(\frac{\gamma A2_{Si,i}}{PMROW_{Si}}\right)^{\sigma A_{Si,i}} \cdot \left(P_{Si,i}\right)^{\sigma A_{Si,i}} \cdot \left(aA_{Si,i}\right)^{(\sigma A_{Si,i}-1)} \qquad (20)$$

$$ET = \frac{1}{INDEXE}\sum\nolimits_{Si}\sum\nolimits_{i}\left(EEU25_{Si,i}\cdot PDC_{Si,i} + EROW_{Si,i}\cdot PDC_{Si,i}\right) \tag{21}$$

$$MT = \frac{1}{INDEXM}\sum\nolimits_{Si}\sum\nolimits_{i}\left(PMEU25_{Si}\cdot MEU25_{Si,i} + PMROW_{Si}\cdot MROW_{Si,i}\right) \tag{22}$$

$$IT = S + SEU25\cdot ER + SROW\cdot ER - \sum\nolimits_{Si}\sum\nolimits_{i}\left(SV_{Si,i}\cdot P_{Si,i}\right) \tag{23}$$

$$\begin{aligned} K_{Si,i} = {} & KL_{Si,i}\left(\frac{\gamma K_{Si,i}}{\left(1+tk_{Si}\right)\cdot RK_{Si,i}+\delta_{Si,i}\cdot PI}\right)^{\sigma KL_{Si,i}}\cdot\left(PKL\right)^{\sigma_{Si,i}} \\ & \cdot\left(aKL_{Si,i}\right)^{\left(\sigma KL_{Si,i}-1\right)} + NF_{Si,i}\cdot fcK_{Si,i} \end{aligned} \tag{24}$$

$$\begin{aligned} L_{Si,i} = {} & KL_{Si,i}\left(\frac{\gamma L_{Si,i}}{PL_{Si}\cdot\left(1+tl1_{Si}+\left(1+tl1_{Si}\right)\cdot tl_{Si}\right)}\right)^{\sigma KL_{Si,i}}\cdot\left(PKL\right)^{\sigma_{Si,i}} \\ & \cdot\left(aKL_{Si,i}\right)^{\left(\sigma KL_{Si,i}-1\right)} + NF_{Si,i}\cdot fcL_{Si,i} + \sum\nolimits_{j}\left(BTRIPS_{Si,i,j}\cdot BTIME_{Si,i,j}\right) \end{aligned} \tag{25}$$

$$\begin{aligned} BTRIPS_{Si,i,j} = {} & BTRIPST_{Si,i} \\ & \cdot\frac{\alpha BT_{Si,i,j}\cdot BTSHARE_{i,j}\cdot\exp\left(-\left(BMONT_{i,j}+BTIME_{i,j}\right)\right)}{\sum_{k}\alpha BT_{Si,i,k}\cdot BTSHARE_{i,k}\cdot\exp\left(-\left(BMONT_{i,k}+BTIME_{i,k}\right)\right)} \end{aligned} \tag{26}$$

$$BRTRIPST_{Si,i} = \beta BT_{Si,i}\cdot NF_{Si,i} \tag{27}$$

$$BTSHARE_{j,i} = \frac{\sum_{Sj}\left(XDDE_{Sj,j,i}+XDDE_{Sj,i,j}\right)}{\sum_{Sj}\sum_{k}\left(XDDE_{Sj,j,k}+XDDE_{Sj,k,j}\right)} \tag{28}$$

$$\begin{aligned} LS_{i} = {} & LS_{i}^{0} + \sum_{j}\left(LMIG_{j,i} - LMIG_{i,j}\right) \\ & - \sum\nolimits_{j}\left(Ttime_{i,j}\cdot LCM_{i,j}\right) \end{aligned} \tag{29}$$

$$\begin{aligned}&P_{Si,i}\cdot(1-sc_{Si}+tc_{Si}+vatc_{Si}+exst_{Si})\cdot C_{Si,i}=P_{Si,i}\cdot(1-sc_{Si}+tc_{Si}+vatc_{Si}+exst_{Si})\\&\cdot\mu H_{Si,i}+\alpha H_{Si,i}\cdot\Big(CBUD_i-\sum\nolimits_{Sj}\big(\mu H_{Sj,i}\cdot P_{Sj,i}\cdot(1-sc_{Sj}+tc_{Sj}+vatc_{Sj}+exst_{Sj})\big)\Big)\end{aligned}\tag{30}$$

$$\begin{aligned}&CBUD_i=Y_i\cdot(1-ty)+TRF_i\cdot GDPDEF+UNEMP_i\cdot PW_i\cdot trep_i-SH_i\\&-\sum\nolimits_j\big(Tmoney_{i,j}\cdot LCM_{i,j}+SHOPTRIPS_{i,j}\cdot SHOPMONT_{i,j}\\&+OTHTRIPS_{i,j}\cdot OTHMONT_{i,j}+EDUTRIPS_{i,j}\cdot EDUMONT_{i,j}\big)\\&\cdot\sum\nolimits_{Sj=transpSj}\big(P_{Sj,i}\cdot(1-sc_{Sj}+tc_{Sj}+vatc_{Sj}+exst_{Sj})\big)\end{aligned}\tag{31}$$

$$Y_i=(LS_i-UNEMP_i)\cdot PW_i+\sum\nolimits_{Si}(K_{Si,i}\cdot RK_{Si,i})-LROW_i\cdot PLROW\cdot ER\tag{32}$$

$$SH_i=mps_i\cdot(Y_i\cdot(1-ty)+TRF_i\cdot GDPDEF+UNEMP_i\cdot PW_i\cdot trep_i)\tag{33}$$

$$S=\sum\nolimits_i(SH_i)+SG\cdot GDPDEF+\sum\nolimits_{Si}\sum\nolimits_i(delta_{Si,i}\cdot K_{Si,i}\cdot PI)\tag{34}$$

$$I_{Si,i}\cdot P_{Si,i}\cdot(1-sc_{Si}+tc_{Si}+vatc_{Si}+exst_{Si})=\alpha I_{Si,i}\cdot IT\tag{35}$$

$$\begin{aligned}&CG_{Si,i}\cdot P_{Si,i}\cdot(1-sc_i+tc_i+vatc_i+exst_i)=\alpha G_{Si,i}\cdot(TAXR-SUBS\\&-\sum\nolimits_j(TRF_j\cdot GDPDEF+UNEMP_j\cdot PW_j\cdot trep_j)-SG\cdot GDPDEF+TREU25\cdot ER)\end{aligned}\tag{36}$$

$$\begin{aligned}&TAXR=\sum\nolimits_i\Big(\sum\nolimits_{Si=XDZSii}\big(PL_i\cdot L_{Si,i}\cdot(tl1_i+(1+tl1_i)\cdot tl1_i+tk_{Si}\cdot K_{Si,i}\cdot RK_{Si,i}\\&+txd_{Si}\cdot XD_{Si,i}\cdot PD_{Si,i}\big)+\sum\nolimits_{Si}\big((tc_{Si}+vatc_{Si}+exst_{Si})\cdot P_{Si,i}\cdot\big(C_{Si,i}+\sum\nolimits_j\big(Tmoney_{i,j}\\&\cdot LCM_{i,j}+SHOPTRIPS_{i,j}\cdot SHOPMONT_{i,j}+OTHTRIPS_{i,j}\cdot OTHMONT_{i,j}\\&+EDUTRIPS_{i,j}\cdot EDUMONT_{i,j}\big)\$transp(\mathrm{sec})+I_{Si,i}+CG_{Si,i}\big)\big)+ty\cdot Y_i\Big)\end{aligned}\tag{37}$$

$$\begin{aligned}&SUBS=\sum\nolimits_i\Big(\sum\nolimits_{Si=XDZSii}(sp_{Si}\cdot XD_{Si,i}\cdot PD_{Si,i})+\sum\nolimits_{Si}\big(sc_{Si}\cdot P_{Si,i}\cdot\big(C_{Si,i}+\sum\nolimits_j\big(Tmoney_{i,j}\\&\cdot LCM_{i,j}+SHOPTRIPS_{i,j}\cdot SHOPMONT_{i,j}+OTHTRIPS_{i,j}\cdot OTHMONT_{i,j}\\&+EDUTRIPS_{i,j}\cdot EDUMONT_{i,j}\big)\$transp(\mathrm{sec})+I_{Si,i}+CG_{Si,i}\big)\big)\Big)\end{aligned}\tag{38}$$

$$\begin{aligned}GDP = {} & \sum_{Si=XDZSii}\sum_{i=XDZSii}\left(XD_{Si,i}\cdot PD^{0}_{Si,i}\right)-\sum_{Sj}\sum_{Si}\sum_{i}\left(io_{Sj,Si,i}\cdot XD_{Si,i}\cdot P^{0}_{Sj}\right)\\ & -\sum_{j}\sum_{i}\sum_{Si}\left(BTRIPS_{Si,j,i}\cdot BMONT_{j,i}\cdot\sum_{Sj=transpSj}\left(P^{0}_{Sj,i}\right)\right)\\ & +\sum_{Si}\sum_{i}\left(\left(tc_{Si}+vatc_{Si}+exst_{Si}-sc_{Si}\right)\cdot P^{0}_{Si,i}\cdot\left(C_{Si,i}+\sum_{j}\left(Tmoney_{i,j}\right.\right.\right.\\ & \cdot LCM_{i,j}+SHOPTRIPS_{i,j}\cdot SHOPMONT_{i,j}+OTHTRIPS_{i,j}\cdot OTHMONT_{i,j}\\ & \left.\left.\left.+EDUTRIPS_{i,j}\cdot EDUMONT_{i,j}\right)\$transp(\sec)+I_{Si,i}+CG_{Si,i}\right)\right)\end{aligned}\tag{39}$$

$$\begin{aligned}GDPC = {} & \sum_{Si=XDZSii}\sum_{i=XDZSii}\left(XD_{Si,i}\cdot P_{Si,i}\right)-\sum_{Sj}\sum_{Si}\sum_{i}\left(io_{Sj,Si,i}\cdot XD_{Si,i}\cdot P_{Sj,i}\right)\\ & -\sum_{j}\sum_{i}\sum_{Si}\left(BTRIPS_{Si,j,i}\cdot BMONT_{j,i}\cdot\sum_{Sj=transpSj}\left(P_{Sj,i}\right)\right)\\ & +\sum_{Si}\sum_{i}\left(\left(tc_{Si}+vatc_{Si}+exst_{Si}-sc_{Si}\right)\cdot P_{Si,i}\cdot\left(C_{Si,i}+\sum_{j}\left(Tmoney_{i,j}\right.\right.\right.\\ & \cdot LCM_{i,j}+SHOPTRIPS_{i,j}\cdot SHOPMONT_{i,j}+OTHTRIPS_{i,j}\cdot OTHMONT_{i,j}\\ & \left.\left.\left.+EDUTRIPS_{i,j}\cdot EDUMONT_{i,j}\right)\$transp(\sec)+I_{Si,i}+CG_{Si,i}\right)\right)\end{aligned}\tag{40}$$

$$GDPDEF=\frac{GDPC}{GDP}\tag{41}$$

$$INDEXE=\frac{\sum_{Si}\sum_{i}\left(EEU25_{Si,i}\cdot PDC_{Si,i}+EROW_{Si,i}\cdot PDC_{Si,i}\right)}{\sum_{Si}\sum_{i}\left(EEU25_{Si,i}\cdot PDC^{0}_{Si,i}+EROW_{Si,i}\cdot PDC^{0}_{Si,i}\right)}\tag{42}$$

$$INDEXM=\frac{\sum_{Si}\sum_{i}\left(PMEU25_{Si}\cdot MEU25^{0}_{Si,i}+PMROW_{Si}\cdot MROW^{0}_{Si,i}\right)}{\sum_{Si}\sum_{i}\left(PMEU25^{0}_{Si}\cdot MEU25^{0}_{Si,i}+PMROW^{0}_{Si}\cdot MROW^{0}_{Si,i}\right)}\tag{43}$$

$$PTM=\sum_{Si}\sum_{i}\left(atm_{Si,i}\cdot P_{Si,i}\right)\tag{44}$$

$$PEV_{i}=\prod_{Si=CZSii}\left(\left(P_{Si,i}\cdot\left(1-sc_{Si}+tc_{Si}+vatc_{Si}+exst_{Si}\right)\right)^{\alpha H_{Si,i}}\right)\tag{45}$$

$$U_{i}=\prod_{Si}\left(C_{Si,i}-\mu H_{Si,i}\right)^{\alpha H_{Si,i}}\tag{46}$$

$$SII_{i}=CBUD_{i}-\sum_{Si=CZSii}\left(\mu H_{Si,i}\cdot P_{Si,i}\cdot\left(1-sc_{Si}+tc_{Si}+vatc_{Si}+exst_{Si}\right)\right)\tag{47}$$

$$EV_i = \left( \left( \frac{PEV_i^0}{PEV_i} \right) \cdot SII_i - SID_i^0 \right) \cdot \frac{1}{Y_i} \cdot 100 \tag{48}$$

$$SV_{Si,i} = svs_{Si,i} \cdot X_{Si,i} \tag{49}$$

$$\begin{aligned} & NF_{Si,i} \cdot elas\mathrm{Re}\,g_{Si,i} \cdot (fcL_{Si,i} + \frac{1}{NF_{Si,i}} \cdot \sum_j \; (BTRIPS_{Si,i,j} \cdot BTIME_{i,j}) \\ & + fcK_{Si,i}) \cdot INDEX_i = XD_{Si,i} \cdot PD_{Si,i} \end{aligned} \tag{50}$$

$$AUXV_{Si,i} = \left( NF_{Si,i} \right)^{\frac{1}{(1 - elas\mathrm{Re}\,g_{Si,i})}} \tag{51}$$

$$\sum\nolimits_{Si} \left( L_{Si,i} \right) = \sum\nolimits_{j} \left( LCM_{i,j} \right) \tag{52}$$

$$PW_i \cdot \left( LS_i - UNEMP_i \right) = \sum\nolimits_j \left( LCM_{j,i} \cdot PL_j \right) \tag{53}$$

$$\begin{aligned} & LCM_{j,i} = aM_{j,i} \cdot \left( LS_i - UNEMP_i \right)^{\alpha M_{j,i}} \cdot \left( \sum\nolimits_{Si} \left( L_{Si,j} \right) \right)^{\left(1 - \alpha M_{j,i}\right)} \\ & \cdot \exp\left( - \beta T_{j,i} \cdot \left( Ttime_{j,i} + Tmoney_{j,i} \right) \right) \end{aligned} \tag{54}$$

$$\log\left( \frac{PW_i}{INDEX_i} \right) = ParW0_i + ParW1 \cdot \log\left( \frac{\sum_{Si} XD_{Si,i}}{\sum_{Si} L_{Si,i}} \right) + ParW3 \cdot \log\left( 1 - \frac{UNEMP_i}{LS_i} \right) \tag{55}$$

$$UNEMP_i = LS_i \cdot UNRATE_i \tag{56}$$

$$\begin{aligned} & SHOPTRIPS_{i,j} = \gamma SHOP_i \cdot \frac{Y_i}{INDEX_i} \\ & \cdot \frac{\beta SHOP_{i,j} \cdot \sum_{Si=shopSi} \left( XD_{Si,j} \right) \cdot \exp\left( - \left( SHOPTIME_{i,j}^0 + SHOPMONT_{i,j}^0 \right) \right)}{\sum_k \left( \beta SHOP_{i,k} \cdot \sum_{Si=shopSi} \left( XD_{Si,k} \right) \cdot \exp\left( - \left( SHOPTIME_{i,k}^0 + SHOPMONT_{i,k}^0 \right) \right) \right)} \end{aligned} \tag{57}$$

$$
\begin{aligned}
EDUTRIPS_{i,j} &= \gamma EDU_i \cdot LS_i \\
&\cdot \frac{\beta EDU_{i,j} \cdot \sum_{Si=edu}\left(XD_{Si,j}\right) \cdot \exp\left(-\left(EDUTIME^{0}_{i,j} + EDUMONT^{0}_{i,j}\right)\right)}{\sum_k \left(\beta EDU_{i,k} \cdot \sum_{Si=edu_{Si}}\left(XD_{Si,k}\right) \cdot \exp\left(-\left(EDUTIME^{0}_{i,k} + EDUMONT^{0}_{i,k}\right)\right)\right)}
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
OTHTRIPS_{i,j} &= \gamma OTHER_i \cdot \frac{Y_i}{INDEX_i} \\
&\cdot \frac{\beta OTHER_{i,j} \cdot \sum_{Si=travelSi}\left(XD_{Si,j}\right) \cdot \exp\left(-\left(OTHTIME^{0}_{i,j} + OTHMONT^{0}_{i,j}\right)\right)}{\sum_k \left(\beta OTHER_{i,k} \cdot \sum_{Si=travelSi}\left(XD_{Si,k}\right) \cdot \exp\left(-\left(OTHTIME^{0}_{i,k} + OTHMONT^{0}_{i,k}\right)\right)\right)}
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
LMIG_{i,j} &= \sum_k LS_k \cdot (Bmig_i + \sum_k U_k / N - U_i) / \sum_l (Bmig_l + \sum_k U_k / N - U_l) \\
&\cdot (Amig_{i,j} + U_j) / \sum_k (Amig_{i,k} + U_k)
\end{aligned}
\tag{60}
$$

$$
KL_{Si,i} = ioKL_{Si,i} \cdot XD_{Si,i} \tag{61}
$$

$$
\begin{aligned}
PKL_{Si,i} \cdot KL_{Si,i} &= \left( K_{Si,i} - NF_{Si,i} \cdot fcK_{Si,i} \right) \cdot \left(\left(1 + tk_{Si}\right) \cdot RK_{Si,i} + \delta_{Si,i} PI\right) \\
&+ PL_i \cdot \left( L_{Si,i} - NF_{Si,i} \cdot fcL_{Si,i} + \sum_j \left(BTRIPS_{Si,i,j} \cdot BTIME_{Si,i,j}\right) \right) \\
&\cdot \left(1 + tl1_{Si} + \left(1 + tl1_{Si}\right) \cdot tl_{Si}\right)
\end{aligned}
\tag{62}
$$